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New Developments in Simplified Control

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ABSTRACT

Problems of control are presented from a standpoint of simple operation rather than mere mechanical simplicity. Of first importance is the elimination of inconsistent or abnormal behavior in which spinning is classed. The influence of wing and engine position is discussed, and several advantages of an automobile-type wheel arrangement are brought out.

A primary control, in which flaps are coordinated with elevator and rudder with ailerons, is advocated. In a possible supplementary pedal control, ground brakes are coordinated with air drag obtained by opposite deflection of two rudder surfaces.

Special problems, considered from a standpoint of simplified control, include stability against overturning on the ground, inertia loads on the wing, spiral stability, and control requirements for following a compass course.

The ideal airplane for private flying is described as being: outstanding in vision, incapable of spinning, comparable with an automobile in simplicity of control, yet with unquestioned superiority of cross-country performance.

SIMPLE CONTROL, from the pilot's standpoint, must have its foundation in positive and consistent relations between the control application and its result. This requirement bans any abnormal flight condition which, even for a brief interval, amounts to loss of control—like the stalled landing still so commonly in use. The development of a more consistent landing procedure, although started where most important, with private planes, has now been applied to almost all types. Other improvements will not long be denied. For example, instead of studying spins as normal or abnormal, the broader and sounder view is now being adopted—that any spin is abnormal and hence should be outlawed from the physically possible maneuvers of any airplane designed for transportation.

Although the nonspinning airplane has only lately been officially recognized, it is not particularly new or difficult of attainment. The earliest such plane known to the author which was essentially in this class was the Aeromarine-Klemm.[1] The original design as imported from Germany was so far from being nonspinning that it would, on rather slight provocation, go into a flat spin from which recovery proved impossible. Fortunately, the light wing loading minimized the fatalities from several such occurrences, but still it was no plane to put in the hands of a novice. Hence, the aerodynamic properties were thoroughly revised, the modifications including less taper, a thicker tip section, less control movement and further forward c.g. In the absence at that time of precise knowledge as to the effect of such changes, most of them could be classed as mere engineering judgment, and at least one—the reduction of taper—was quite fortuitous. The result was most gratifying. After test pilots were unable to spin the plane, the air commerce inspector finally succeeded, by a combination of whip stall and crossing of controls, to hold about two turns, after which it would recover by itself. This result seemed even better than being totally unable to spin, because the tradition was then quite strong that when a ship was hard to spin it was also difficult to pull out.

The next step in the development of simplified control may well be ascribed to the first Stout Sky Car of eleven years ago.[2] Again there was no attempt to make it absolutely nonspinning but merely spin resistant to the extent that it could not fall into a spin inadvertently from any conceivable attitude of normal flight. It had essentially the modern tricycle landing gear, except for the steerable front wheel, and was also designed for what is now known as two-control operation (although the rudder pedals were retained for those who insisted on using them). As originally assembled with the floating wing-tip ailerons, it had the most perfect lateral control of any plane the author had ever flown. By overbalancing the ailerons with weight forward they were put normally in a position of slight negative lift, which in turn gave a slight yaw in the favorable sense when operated. As a result, almost any degree of turn could easily be made with a quick entry and a perfect bank by using the stick alone and with feet entirely off the pedals.

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Either of the above-mentioned aircraft, and doubtless others as well, would have been completely spin-proof if independent rudder control had been eliminated.

Further developments along similar lines have included the Weick W-1, the Hammond Y, the Gwinn Aircar, the Ercoupe and the General Skyfarer, to name only those airplanes in the unconventional class designed for private use. Most of these have shown an unmistakable trend toward spin-proof qualities, the level-landing type of wheel arrangement, front wheel steerable by the control wheel, two-control operation and a somewhat hesitant use of flaps. Questions that are still unsettled are: pusher versus tractor, high-wing versus low-wing, important features of the landing gear, and specific means of operating and coordinating the various controls.

Engine and Wing Position

Although the position of the engine and wing is usually dictated by considerations other than control, the following control factors should be kept in mind:

The pusher engine involves more shift of center of gravity as between full load and light load, but, on the other hand, can be arranged to give less change of trim than a tractor as between power on and power off. One reason is that the pusher engine can be mounted considerably higher, thereby helping to counteract the usual nosing up tendency when power is applied. Another is that more effect can be obtained by a change in slope of the thrust line in a vertical plane, this being of relatively minor effect in a tractor because of the predominating influence of the wing on the direction of the slip-stream.

Maintenance of the same stalling characteristics for power on or off is also a distinct contribution of the pusher engine to simple, consistent control. Although the slipstream effect on the wing lift is thereby lost, the time when such help would be most welcome is usually when it is nonexistent.

The main advantage of the pusher is, of course, vision, the all-important reference on which any control must normally depend. The pusher arrangement is particularly desirable if a high wing is used.

As is well known, the high-wing position makes both longitudinal and lateral stability easy to obtain, especially at the more critical high angles of attack, leading to a type of control that is generally more positive and consistent. This does not mean that simple and satisfactory control cannot be had with a low-wing ship but only that such a case is the exception rather than the rule.

Tricycle Landing Gear

As the landing and, to a lesser extent, take-off have hitherto been the most critical parts of a pilot's primary education, the landing gear calls for first attention in the discussion of control means. Without going into the now well-recognized advantages of the so-called tricycle gear,[3] certain faults in its operation, and whether there is not still room for considerable improvement, should be considered.

One criticism that has been seriously made of the tricycle gear is this: Any pilot placed in such a plane, even though experienced and generally sensible, after a period of somewhat timidly feeling out its characteristics and satisfying himself that it can be landed at the will of the pilot instead of at the will of the plane, will adopt the attitude that he no longer needs to be careful. The plane will be landed with little regard for the wind, with practically any speed of descent, and using the brakes without mercy until something gives way. This indicates the need for unusual attention to strength factors and brake capacity if the inherent advantage made mechanically possible by the tricycle gear is to be largely realized. With respect to side-wind landings, there is, at the same time, a rather basic factor that has apparently not yet been given adequate attention.

A primary condition for simple consistent control is obviously that the control movement in a given direction must always serve a similar function. Examples of inconsistent control, familiar to any pilot of a conventional plane, include pushing forward on the control when the nose goes down in a spin, holding the control back at the moment of landing, and the formerly common practice of moving the aileron control away from the side toward which it is desired to turn on the ground. The first example is eliminated in the nonspinning plane; the second, by the tricycle or level landing gear; while it might be supposed that the third would be eliminated by the steerable front wheel geared to the control wheel. Unfortunately, however, conditions are thereby introduced which may make the aileron control still less consistent.

Steerable Front Wheel

Whether two-control or three-control operation is used on the immediate approach to a landing, the wings must be brought to a reasonable level by use of the control wheel. Suppose that as a result of a cross-wind landing or offset position of the control, one wing persists in going up after ground contact has been made. To be entirely consistent, the pilot would then attempt to lower the high wing by rotation of the control wheel in that direction. This, however, turns the front wheel now in contact with the ground, in a direction to throw the centrifugal force the other way, which invariably overbalances the aerodynamic effect of the ailerons and, instead of correcting the rolling tendency, actually makes it worse.

Consider the example of a pilot approaching for a landing and discovering that he is drifting laterally as he nears the ground. If there is still room for a banked turn, he, of course, uses the remaining height to turn as far as he can into the wind. On closer approach to the

ground, however, he brings the wings level, and to stop them in the level condition he reverses again momentarily. In this way actual contact may be made with some of the same lateral drift still remaining and with the front wheel turned somewhat into the wind. This puts the centrifugal force, as well as the primary lateral drift force, in a direction to cause a down-wind overturning tendency, with the resultant inconsistency of control reaction already mentioned. If a wing is damaged it may be called "pilot's error," but the fact remains that there are sure to be plenty of such errors in any airplane that invites them.

A fixed front wheel seems definitely better than one that might be turned wrong. Yet with the steerable front wheel accepted as generally desirable, there are at least three cures for this tendency to steer in the wrong direction: First, let the pilot forget that he is flying an airplane as soon as he makes contact with the ground, and then imagine that he is in an automobile to be controlled accordingly (by steering into the bank). Second, with a suitably designed front wheel, simply release the control entirely and let the dynamics of the situation take charge.[3] Third, provide a wheel spacing that in itself will prevent overturning. Neither of the first two measures is likely to be sufficient in itself because both depend on abnormal action or inaction by the pilot. Even aside from wrong control, the possibility must be considered of the front wheel dropping into a rut or that positive steering must be used to avoid obstructions. Hence it is important to provide enough wheel base and tread to take care of any situation that may be reasonably expected. This condition is particularly critical with the high-wing arrangement because of its relatively high center of gravity and also because the most logical mounting for the wheels in that case is on the fuselage or body.

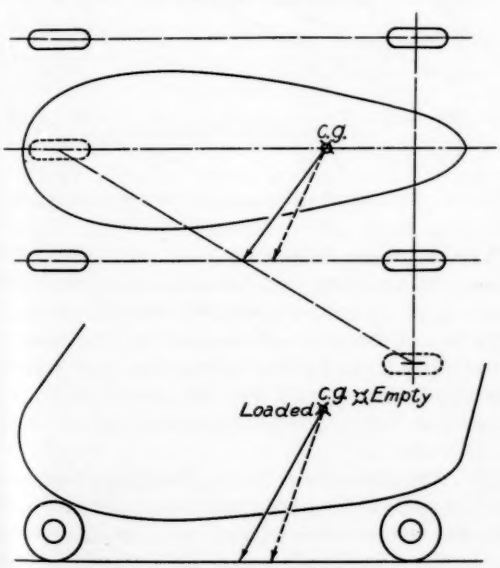

FIG. 1. Landing force diagram.

THREE VERSUS FOUR WHEELS

Fig. 1 shows two views of an airplane body in which the dotted wheels show a tricycle arrangement and the full-lined wheels show the equivalent four-wheel gear. The arrow in each case represents the weight and inertia force projected into the plane of the paper, with the brake-force coefficient 0.55 and the lateral force 0.75 of the vertical component. The solid arrow (0.93 resultant ground force) represents the condition with brakes applied on all wheels; the dotted arrow, with brakes on the rear wheels only. With the scale chosen (which makes the force vector just reach the ground), the limiting condition at overturning is represented by that which makes the point of the arrow in the top view just touch the lateral wheel line. It is evident that with all wheels brake-equipped the rear wheels of a tricycle gear must be placed at an almost prohibitive distance from the body to get the same ground stability against overturning as is provided by the four wheels shown, the latter having no greater tread width than that of an automobile. The total brake force obtainable from all wheels is over 60 per cent greater than with brakes on the rear wheels alone, the relative stopping distance being approximately in inverse proportion. While there is some question as to the validity of starting with the two ground-force components rather than with the magnitude of the force itself and its longitudinal component, the conclusions are qualitatively the same in either case. The term "ground force" refers to any force in a horizontal plane transmitted by the airplane to the ground.

Contrary to automobile practice, it seems desirable to give the front wheels a tendency to lock slightly before the rear wheels, although the locking of any wheels should be possible only with a very substantial pedal force. In general, the ground stabilizing factors are chosen with a view of resisting a turn rather than accentuating it and without much concern for tire wear, which is normally of small importance in an airplane. It may be remarked in passing that the commonly used circumferential ribs on airplane tires are quite unsuited to either type of landing gear. Whereas an important object in an automobile is to *prevent* skidding, the object in an airplane should be to *permit* ground skidding, particularly of the front wheels, at any lateral force approaching a critical value.

Because of the offset position of the kingpin pivot axis for each of the two front wheels of the four-wheel gear, there are other important geometric variables in the linkage besides the caster angle utilized in the single front wheel. For example, proportions can be readily chosen such that the stabilizing tendency of the wheels will be accentuated by brake application or soft ground. For means of avoiding "shimmy," there is a wealth of experience in automobile practice. In a high-wing airplane of a size permitting a choice of suitable wheels, there is little drag or weight difference between the

three and four-wheel gear; the latter affords an appreciable margin of security against the failure of a wheel or tire, particularly in front.

Advantages and special characteristics of the four-wheel gear have been brought out not by way of arguing its adoption in all cases but to show what can be done with an alternative arrangement. In actual practice, the number of wheels, like the position of the wing, must depend largely on considerations of purpose and general arrangement.

Position of Rear Wheels

If the tail is to be designed for no ground contact and if the plane in general is to resist a reasonable ground wind without staking down, it is important to put the rear wheels a substantial distance behind the most rearward c.g. To a certain extent this requirement fits in with the desire to prevent taking off prematurely. Yet there must be no interference with getting a sufficient angle of attack for a proper take-off at a speed preferably about 20 per cent above stalling. The combined requirements are most difficult of attainment for a high-wing pusher because of the high center of drag and weight and the large rearward shift of the c.g. on removal of disposable load, not to mention the thrust force. The original Sky Car, for example, would immediately tip back on its tail when the occupants got out. This action was taken care of by having a tail wheel in addition to the nose wheel, but at the best it was disconcerting to a passenger. At the worst it made the ground attitude of the empty ship no better than with a conventional landing gear as far as reaction to ground winds was concerned.

If at the moment of take-off the effective angle of attack can be increased sufficiently by flaps, it is obvious that a take-off can be made with the rear wheels in almost any position. But, although flaps are useful in take-off for other reasons as well, the use of a separate flap control for the purpose would be anything but a simplification. The solution of this problem can be combined with other desired results in the method of coordinating the longitudinal control.

Lift Control

The primary lift control has always been, and will continue to be, the fore-and-aft movement of the control column, traditionally through the medium of the elevator. The only other instrumentality involving lift in any major sense is the flap. Hitherto, the flap has usually been operated as a combined lift and drag control by means of a separate flap control lever. Such an arrangement is admittedly simple mechanically but complicated in operation, since operating simplicity requires coordination of like rather than unlike functions. Hence, with a relatively low-drag flap[7] (of which there are several types now available), the rational place for its control is in direct coordination with the elevator.

In order to minimize for the normal cruising range such drag as is still unavoidably involved in flap operation, the flap movement should be designed to pick up in parabolic relationship to the elevator as the control column is pulled back. The effect on the wing section characteristics of the combined control in comparison with the normal elevator is shown in Fig. 2. In the lower part of this figure the heavy full lines show the increase of lift coefficient with angle of attack of the basic airfoil (and of the airplane bodily) as produced by operation of the elevator alone. The heavy dashed line cutting across the lift curves for varying flap deflection shows the effect of the proposed flap and elevator combination. The corresponding drag is shown above.

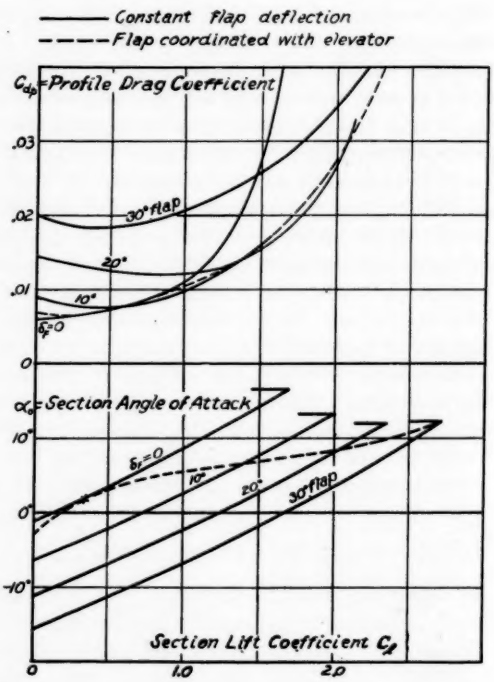

Fig. 2. Example of coordinated lift with incidental drag.

With a proper linkage ratio in the system, there appears no reason to expect an essential operating difference in the process of getting lift whether by angular deflection of the entire airplane or by combining this process with that of deflecting the wing or a part of it. The detail characteristics of the coordinated control are subject, however, to a few qualifications that include the following:

(1) The direct use of the flap facilitates take-off not only by reason of the lift itself (which can be withheld until the moment when it is needed) but by permitting the application of substantial lift without depressing the tail, as already mentioned.

(2) Clearing an obstacle is made somewhat more positive by the larger lift quickly available, as well as by the fact that for a given lift the tail need not be depressed so much.

(3) For given balance characteristics, the control forces are increased by an increment that is desirable in the smaller planes, but in much larger sizes and higher speeds will undoubtedly set a somewhat lower limit to what can be handled without boosters or servos.

(4) There will be more aerodynamic margin of safe control in maneuvers such as sharp turns and pull-ups because, for a given air attitude of the airplane, a greater effective camber of the wing is automatically provided to carry the load put on it.

(5) The control reaction will be so direct and unescapable that there would appear to be little or no excuse for inadvertent mistakes such as too suddenly retracting the flaps. It must be expected, however, that the airplane lift will always react more quickly to the flap than to elevator deflection.

(6) For the wing stress analysis, instead of considering flaps-down as a supplementary condition, each major condition is governed by its own flap deflection.

(7) A maneuvering load factor can be realized more quickly but with a more substantial control force, so that on the whole there would appear less reason for inadvertent overloading.

(8) Landing loads should be appreciably less because of the facility of quickly checking the sinking speed without waiting for the tail to drop. Like any other plane, however, the glide-in should never be made at the rearward control limit.

The feasibility of gliding in with a substantial speed reserve, at the same time maintaining a steep approach, depends, of course, on drag as well as lift.

Drag Control

There is no longer much room for argument on the desirability of separating drag as far as possible from lift and employing it as a secondary control.[4] The point is well realized by any pilot who has dropped down toward a small field with split flaps set at a large angle only to find, on close approach, that the field is unusable and that he must climb out again.

With the object of utilizing other existing surfaces and controls if possible, the following appear to be the most plausible means of effecting the desired functional separation, alternatively:

(1) Design the flap so that beyond the lift range there is a range of sharply varying drag but practically constant lift.[3] This part of the movement obviously should not be kept on the main control column but might be put on a separate lever. This still, however, leaves several apparent chances for confusion of control.

(2) Use two rudders or a split single rudder so that the two surfaces may be deflected outward by pressure on a foot pedal. There must be at least one pedal for brake operation on the ground, and it can be used for the aerodynamic brake as well. The use of the rudders provides ample drag for glide path control (up to a gliding angle of about 1:7); they can be readily arranged to retain a differential movement for lateral control purposes and, by their position at the rear, give an added stabilizing effect both directionally and longitudinally.

In a broad sense, directional control can be considered as differential drag; hence, any ship for which some measure of independent directional control is desired can be equipped with the conventional two pedals instead of one and can be arranged so that the depression of one or the other pedal turns one or both rudders in the corresponding direction. The application of a symmetric drag force will then be by depressing both pedals. Somewhat similar independent action of the two pedals has been used before (notably on tailless airplanes) and was tried out by the author several years ago with complete success. The fear expressed by some that there might be danger of inadvertently pressing on both pedals at an inappropriate time, such as during the take-off, did not seem to be borne out in practice. In fact one flight instructor thought the possibility of such an occurrence was a distinct training asset because it would so positively discourage undue pressure and mental tension on the part of a student.

For most purposes, of private flying at least, it appears preferable to stick to the single pedal and coordinate the usual rudder control entirely with the ailerons. In the interest of functional separation, it is important to position the rudder surfaces in such a way that their deflection will have no pitching effect.

Steering Control

Coordinated aileron and steering control is, of course, not essentially new. The original Wright airplane employed it in principle, and almost any well-designed modern plane can be steered in a reasonably safe manner on the ailerons alone. There appear to be good reasons, however, for designing a definite two-control airplane on the basis of direct coordination of actual rudders with the aileron system (as did the Wrights) rather than by relying on sideslip for supplying the necessary lateral tail reaction.

Two-control ships hitherto available have called forth considerable criticism from trained pilots who did not feel at home in a plane that yawed and slipped perceptibly and yet which gave them no means by which it could be remedied. Such opinions, based on flight habits in a conventional type of airplane, mean nothing in themselves—at least from the standpoint of safe control. The danger of a slip, and particularly of a skid, has nothing directly to do with the maneuver itself but only with the dangerous attitude of the airplane that such a maneuver may lead to; any plane that is prevented by other means from staying out of a spin is still safe no matter how much it may slip or skid in the turn. Such criticism as the above has usually been applied to

relatively sharp turns where it is really not warranted. Experience, however, has indicated a real objection to departures from a perfect bank as applied to slight turns.

This objection has to do with the procedure of following a compass course for which the flight path, instead of being straight, is in reality a series of flat curves. A plane that depends on slipping for its directional control takes an appreciable time to swing into the turn after banking, and it similarly skids and wants to keep turning on the recovery. Hence, the tendency is for the plane to be late going in and late again coming out so that it turns beyond the heading anticipated. It happens that the compass on a southern heading acts just opposite to this tendency and encourages straightening out too soon; hence, on this heading, the control and instrumental tendencies supplement each other, with a result that is usually satisfactory. On a northern heading, on the other hand, the two tendencies add up to a result that is worse than either and which by actual experience with an ordinary type of compass can be so serious as to make difficult the following of a reasonably straight course in even mildly rough air. Experience shows, however, that it is readily possible to make a rudderless airplane behave better than one with improperly coordinated rudders.[5]

As shown by Fig. 3, even differential ailerons give unfavorable yaw moments at small deflections.[6] To remedy this condition, the coordinated rudder movement should be enough to overbalance substantially the aileron yaw, particularly at small deflections, the same as would be done consciously or subconsciously by a good pilot under similar circumstances. The proper magnitude of directional overbalance depends, of course, on the relative rolling velocity and the moment of inertia around the vertical axis. Another consideration is the unfavorable rolling arm of the usual rudder, which should be kept as small as possible.

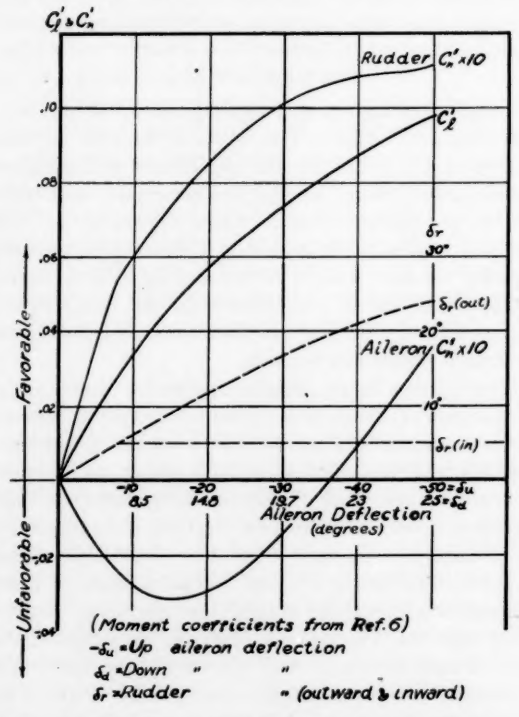

FIG. 3. Example of coordinated steering by two ailerons and two rudders.

SPIRAL STABILITY

A disadvantage of the rudderless airplane is the spiral instability necessarily involved. It is true that probably no airplane flying today is spirally stable to any appreciable or practical extent, regardless of its control system. This condition is not so much inherent, however, as due to the somewhat arbitrary restrictions of dihedral and other characteristics imposed by the conventional control system. For example, aileron control by itself quickly becomes indeterminate, or even has a reverse effect on roll, as the dihedral is increased. With adequate coordinated rudder movement, however, substantial dihedral will help rather than hinder the attainment of the proper bank.

The amount of dihedral necessary for positive spiral stability can be minimized by resort to other expedients such as the introduction of effective fin area forward to supplement a corresponding increase at the tail in resisting yaw and sideslip. One of the best ways of doing this is by use of a compact streamlined body of rounded section such as will give generally efficient performance in a pusher type of plane.[8] Such a body has its lateral center of pressure far forward so that it is equivalent to a fin surface considerably forward of the nose end. Another possibility of promoting spiral stability is by direct damping of turn by drag elements near the wing tips which will automatically function by differential tip speed.

Any model builder knows that it is possible to achieve spiral stability with nothing more than adequate dihedral, although this by itself will probably not be an entirely satisfactory means for a practical airplane (largely due to the effect of side gusts).[9] Still, with so many possibilities available the author does not fall in with the commonly accepted attitude that substantial spiral stability is an impractical dream. The solution of this problem by simple means would at one stroke remove the major difficulty and danger from blind flying. A good illustration of this fact is that airships of the lighter-than-air type have never imposed any problem of control in blind flying. The obvious reason is that they are spirally stable, even though less stable than most airplanes in a purely directional sense.

CONCLUSIONS

Although it is believed that many of the features described can be adopted advantageously for other types (certainly including the military), they are most di-

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Isaac Newton and Aerodynamics

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Introduction

On Christmas day of the year 1642 in a manor house in Woolsthorpe by Colsterworth in Lincolnshire, England, a son was born to Hannah Newton, young widow of the farmer Isaac Newton, who died a few months before the birth of his son. Historians and biographers have tried in vain to find any person of unusual brain power or unusually strong personality in the ancestry of this child. His father was "a wild, extravagant and weak man"—at least according to testimony of his stepfather, the Reverend Barnabas Smith, who married Hannah Newton a few years after the death of her first husband. The boy himself was prematurely born and so weak that he was not expected to survive. However, he lived and became one of the greatest scientific minds of all time, the man of whom Alexander Pope wrote:

> "Nature and Nature's laws lay hid in night.
> God said, Let Newton be!—and all was light."

Mathematicians, physicists and astronomers claim Isaac Newton as their own. However, engineers also have a good claim to a certain portion of his scientific interests and creative ideas. Back of Queen's College in Cambridge, England, an old wooden bridge spans the Cam River. The story goes on the Cambridge campus that the stress analysis of this bridge was worked out by Newton, who was Lucasian Professor of Mathematics at the University. The influence of Newton's conceptions of dynamics was so far reaching that it would be difficult to follow up all of the contributions of the Newtonian Mechanics to engineering and engineering science. However, aeronautical engineers might be interested in his contributions to fluid mechanics and especially to the problem of air resistance.

Laws of Fluid Motion

The first systematic treatment of the equations of fluid motion was given by Euler about a half century after Newton's work. Newton's methods were speculative and empirical rather than strictly logical and analytical. For example, the first edition of his *Mathematical Principles of Natural Philosophy* contains the erroneous statement that the velocity of a fluid issuing through a small orifice in the bottom of a vessel filled to the height h is equal to $\sqrt{gh}$, although Toricelli before him recognized the fact that the velocity of efflux through an orifice is equal to $\sqrt{2gh}$, i.e., the velocity of free fall of a body starting with zero velocity at the height of the water level. Newton was led to his erroneous statement by observation of the fluid volume leaving the orifice in unit time, which due to the contraction is considerably smaller than the product of the cross-sectional area of the orifice and the velocity $\sqrt{2gh}$. One of his collaborators called his attention to the error, and, reconsidering the case, Newton discovered the *vena contracta* and determined by an approximative theoretic calculation the contraction coefficient as $\sqrt{1/2}$, which is not too far from the actual value.

Air Resistance

Newton's formula for the resistance encountered by a body moving uniformly through a fluid medium was the standard formula for computation of air resistance and wind pressure on structures through more than two centuries. The development of our knowledge of air resistance is certainly one of the interesting chapters in the history of science.

The ancient philosophers—the term philosopher included at that time any scholar of natural sciences—did not have the true concept of air resistance. As a matter of fact, the Aristotelian philosophy and science had the vague idea that the air exerted some force on the projectile which kept it moving. Aristotle himself described this motion of a body in air in a rather interesting way. He believed there could not be any vacuum in nature, so that the vacuum that was created by the displacement of the body was filled by the air streaming into the evacuated space and the inflowing air pushed the body forward. Leonardo da Vinci, who was less of a philosopher than an artist and a keen observer of nature, recognized the existence of fluid resistance and also the vortex formation behind a body as the main source of its resistance. As a matter of fact, he invented streamlining as a method of reduction of resistance long before science justified it.

Galilei, who, for some unknown reason, is called by his first name, Galileo, was the first scientist who clearly separated idealized motion without friction and resistance from actual motion and clearly stated that the effect of the medium is not that of pushing a body but decelerating it if it moves without action of other forces, or decreasing its acceleration if it moves under action of gravity. He made experiments to find the relation between air resistance and velocity; he found in the case of pendulums that "they have been resisted by air pro-

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portionally to their velocities," which is true if the resistance is produced merely by the viscosity of the medium. However, Galileo did not attempt to deduce the law of air resistance from the laws of fluid motion.

Newton's Deduction

Newton's deduction of the law of air resistance is based on correct mechanical principles under the assumptions made by him. He assumed that the major part of fluid resistance is due to the inertia of the fluid matter. Under this assumption he found that the resistance is proportional to the density of the fluid, to the square of the velocity of the body relative to the fluid and the square of the linear dimensions of the body. He clearly stated the law of mechanical similarity corresponding to his assumptions; therefore, he has to be credited with the discovery of the drag formula which is used every day in aerodynamic calculations. He then went further and tried to compute the magnitude of the fluid resistance for bodies of a given shape and also the *optimum ogive*, i.e., the shape for the minimum resistance of a projectile. In these calculations he assumed that the fluid consists of equal and equidistant particles that have no mutual actions whatever. This assumption led him to his erroneous conclusions: first, that the air resistance is determined merely by the shape of the frontal part of the body, because only this part is hit directly by the particles of the fluid; second, that the contribution of an arbitrary surface element depends on its inclination only, being proportional to the square of the sine of the angle of inclination between the element and the direction of motion.

It has often been stated, and it is true to a certain extent, that the common belief in the correctness of Newton's theory of air resistance was an impediment in the solution of the problem of mechanical flight. In fact, the strict application of Newton's theory gave a pessimistic prognosis for the feasibility of practical flying machines. According to this theory the force acting on an inclined surface is produced by inelastic impact of the mass which hits the surface in unit time. This mass depends on the frontal area $S \sin \alpha$ and is equal to $\rho v S \sin \alpha$, where ρ is the density of the air; S, the surface area; v, the relative velocity between the air stream and the surface; and α, the angle of inclination (angle of attack). The change of momentum of the unit air mass is equal to the normal component of the velocity, i.e., to $v \sin \alpha$. Therefore, the normal force is given by

$$N = \rho S v^2 \sin^2 \alpha \qquad (1)$$

Thus the lift is equal to

$$L = N \cos \alpha = \rho S v^2 \sin^2 \alpha \cos \alpha$$

and the drag

$$D = N \sin \alpha = \rho S v^2 \sin^3 \alpha$$

Hence the lift and drag coefficients would be given by

$$C_L = 2 \sin^2 \alpha \cos \alpha \qquad (2)$$

and

$$C_D = 2 \sin^3 \alpha \qquad (3)$$

It is evident that according to this theory fair values of C_L/C_D can be obtained only for small values of α; however, for such values the lift coefficient C_L would be extremely small. For example, if it is assumed that $C_L/C_D = 10$, $C_L = 0.02$. The effect of skin friction being neglected in this calculation makes the conditions even worse.

The theoretic lift coefficient of a plane airfoil of infinite aspect ratio, according to modern theory, is equal to

$$C_L = 2\pi \sin \alpha$$

If it is assumed that the average vertical momentum transferred to a unit air mass involved in the lift producing process is equal to $v \sin \alpha$, the mass of air involved in the process, according to modern theory, is equal to $\rho v c \pi$ per unit span, where c is the chord. This mass corresponds to an air stream of the width $3.14c$, whereas the mass involved in the same process, according to the Newton theory, corresponds to an air stream of the width $c \sin \alpha$. This explains the tremendous discrepancy as far as the magnitude of the lift is concerned.

The discrepancy was discovered by experiments soon after the publication of the Mathematical Principles, and the writer does not believe that the progress of flight was really delayed by Newton's authority. However, Newton's formula for the air pressure on inclined surfaces was repeated in hundreds of books and official specifications. It is believed that the civil engineers were the last ones who recognized the discrepancy between the old theory and the experimental facts. In building codes of several countries, and states and counties the wind pressure on inclined roofs was specified according to Newton's formula as late as the last decade. This is really a remarkable proof of the inertia of official specifications, since, according to experimental evidence and also the modern theory, the wind might exert a lifting force on a roof consisting of two slightly inclined surfaces, whereas Newton's theory yields a downward directed force.

Newton himself was dubious as to how far his results obtained for an idealized fluid medium could be applied to real fluids. It seems he considered the air as "a medium consisting of discrete particles." Then he says: "In continued medium, as water, hot oil, and quicksilver, the obstacle as it passes through them does not immediately strike against all the particles of the fluid that generated the resistance made to it, but presses only the particles that lie next to it, which press the particles beyond, which press other particles, and so on; and in these mediums, the resistance is diminished by one-half."

(*Continued on page 548*)

Proportioning a "Canard" Airplane for Longitudinal Stability and Safety Against Stall

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SUMMARY

The conditions for static longitudinal stability and controllability and for safety against stall of a canard-type airplane are investigated.

It is found that the designer's freedom of initiative in proportioning a canard is restricted in very definite ways. The fact that some of these restrictions have probably not been entirely known may serve to explain some of the failures of the past.

The rules that must be followed in order to secure longitudinal stability and controllability and safety against stall are determined for both the conditions of power-off and power-on, whether the high-lift devices are operating or not.

High-lift devices on the tail only, as they have been used in two recent canards, are found to be dangerous, unless the range of the allowable c.g. locations is greatly reduced. The use of tail high-lift devices when the wing is equipped with flaps is recommended, but it is found that the wing and tail high-lift devices must be operated in conjunction with each other and under a single control. The relation that must exist between their deflections is established.

The procedures for the selection or determination of the airfoil section, aspect ratio and area of the tail surface, tail length and angle of setting of the stabilizer are indicated.

It is felt that the tail-first airplane still deserves the earnest attention of designers.

INTRODUCTION

THE SEVERAL ADVANTAGES that the tail-first or canard-type airplane offers over the conventional airplane have attracted the attention of designers since the earliest days of aviation. However, to date, practically all attempts toward a satisfactory design of a canard have failed. The consequence of this discouraging record has been the total disappearance of it in recent years.

The proportioning of a canard for longitudinal stability and controllability and for safety against stall is quite different from that of a conventional airplane. The occurrence of a spin in a canard would be extremely dangerous; therefore, a canard *must*, and fortunately can, be made safe against stall. On the other hand, such safety cannot be achieved without a sacrifice of the controllability and of the range of the allowable c.g. locations. In order to attain all the advantages the canard can offer, it is necessary to observe a number of strict rules so that little freedom is left to the designer of a canard in proportioning his plane. For instance, the effectiveness of the tail must be kept within two usually close limits: a maximum limit, which is imposed by the condition of static longitudinal stability, and a minimum limit, which is imposed by the condition of longitudinal controllability. Analogous restrictions are found in the range of the allowable c.g. locations, the selection of the aspect ratio and airfoil section of the tail surface, the operation of the high-lift devices, and so on. If such restrictions are not known and observed, a failure is almost inevitable.

It is the purpose of this paper to determine some of the above-mentioned limitations, thereby offering a probable explanation for some of the failures of the past and suggesting a possible remedy for future designs.

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SYMBOLS

A.C.	= aerodynamic center of the wing
c	= mean aerodynamic chord of the wing
S	= area of wing
S_t	= area of tail surface
M	= total pitching moment about the c.g. (positive when stalling)
C	= chordwise force on wing
L	= lift of wing
L_t	= lift of tail
C_{Lt}	= $L_t/(\rho/2)S_tV^2$
M_W	= pitching moment of wing about the A.C.
m	= $dC_L/d\alpha$
m_t	= $dC_{Lt}/d\alpha$
i	= number of propellers

Fig. 1 shows the meaning of other symbols as used in this paper. h, l and j are measured from the A.C., positive when forward; k is measured from the wing chord, positive when the c.g. is above the chord. All other symbols have the standard meaning.

The normal force component on the wing is assumed to be equal to the wing lift.

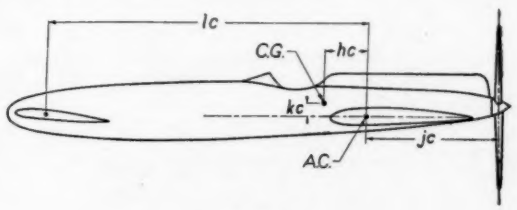

FIG. 1.

The direction of the resultant air reaction on the tail is assumed to be perpendicular to the wing chord and equal to the tail lift. The location of the center of pressure on the tail is assumed to be constant.

The pitching moments contributed by the body, nacelles, fins and rudders and by the propeller thrust are neglected.

The chordwise force coefficient C_c is approximately

$$C_c = \alpha C_L - C_D$$

Write

$$C_D = C_{D_p} + C_{D_i}$$

where C_{D_p} is the parasite drag coefficient, which is here assumed to be constant, and C_{D_i} is the induced drag coefficient:

$$C_{D_i} = \alpha_i C_L$$

where α_i is the induced angle of attack.

Let α_∞ be the angle of attack of the wing measured from zero lift, which would give lift coefficient C_L with two-dimensional flow about the wing profile, and let $-\alpha_0$ be the geometric angle of attack of zero lift. Then

$$\alpha - \alpha_i = \alpha_\infty - \alpha_0$$

whence

$$C_c = (\alpha_\infty - \alpha_0)C_L - C_{D_p}$$

But

$$\alpha_\infty = C_L/m_\infty$$

where m_∞ is the slope of the lift curve of the wing with infinite aspect ratio. Then

$$C_c = (C_L^2/m_\infty) - \alpha_0 C_L - C_{D_p}$$

and

$$dC_c/dC_L = 2\alpha_\infty - \alpha_0$$

The down-wash effect of the tail on the wing is neglected. Therefore

$$dC_{L_t}/dC_L = (dC_{L_t}/d\alpha)/(dC_L/d\alpha) = m_t/m$$

The lifting force acting on the plane of rotation of each propeller when its axis is inclined to the direction of motion is approximately[4, 5, 6]

$$F = C_F \rho S V^2 \alpha_p/2$$

with

$$C_F = K \frac{D^2}{S}\left[\frac{C_Q}{J} - \frac{1}{2}\frac{dC_Q}{dJ}\right]$$

where D is the diameter of the propeller, C_Q is the torque coefficient defined as Torque/$\rho n^2 D^5$, J is the advance-diameter ratio V/nD, α_p is the inclination of the axis to the direction of motion, and K is a coefficient depending on J (Fig. 2).*

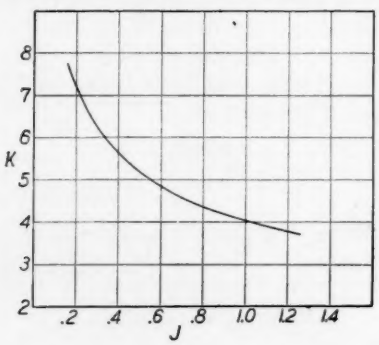

FIG. 2.

STATIC LONGITUDINAL STABILITY AND EQUILIBRIUM

The total moment about the c.g. may be written in the form:

$$M = L_t(l - h)c + M_W + Ckc - Lhc + \sum_i F(j - h)c$$

Dividing throughout by $\rho V^2 Sc/2$,

$$C_m = C_{L_t}(l - h)\frac{S_t}{S} + (C_m)_W + \left(\frac{C_L^2}{m_\infty} - \alpha_0 C_L - C_{D_p}\right)k - C_L h + \sum_i C_F(j - h)\alpha_p \quad (1)$$

The condition for equilibrium at the incidence at which the wing lift coefficient is C_L is $C_m = 0$, whence

$$C_{L_t} = \frac{S}{(l - h)S_t}\left[C_L(h + \alpha_0 k - \alpha_\infty k) + C_{D_p}k - (C_m)_W - \sum_i C_F(j - h)\alpha_p\right] \quad (2)$$

The angle of setting of the tail, or the equivalent elevator deflection, for equilibrium at any given angle of attack of flight and for any given c.g. location can be obtained from Eq. (2). The angle of setting of the tail (angle between the chord of the tail and the chord of the wing) is the difference between the angle of attack of the tail, which gives the tail lift coefficient C_{L_t}, and the angle of attack of the wing, which gives the wing lift coefficient C_L.

From Eq. (1),

$$\frac{dC_m}{dC_L} = \frac{m_t}{m}(l - h)\frac{S_t}{S} + (2\alpha_\infty - \alpha_0)k - h + \frac{1}{m}\sum_i C_F(j - h) \quad (3)$$

For stability, dC_m/dC_L must be negative.

Eq. (3) shows that if k is positive (c.g. above chord) the stability decreases as the angle of attack increases;

* New information on this propeller force has been published recently by Rumph, L. B., White, R. J., and Grumman, H. R., *Propeller Forces Due to Yaw and Their Effect on Airplane Stability*, Journal of the Aeronautical Sciences, Vol. 9, No. 12, pp. 465–470, October, 1942.

if k is negative (c.g. below chord), the stability increases with the angle of attack. It also shows that the stability is greater with power-on if $j - h$ is negative (plane of rotation behind the c.g.) and greater with power-off if $j - h$ is positive.

For neutral stability, dC_m/dC_L must be zero, whence

$$h_0 = \frac{m_t \frac{S_t}{S} l + (2\alpha_\infty - \alpha_0)mk + \sum_i C_F j}{m + m_t \frac{S_t}{S} + \sum_i C_F} \qquad (4)$$

Eq. (4) gives the locus (h_0, k) of neutral c.g. locations, i.e., the locus of the most rearward allowable c.g. locations for stability. For $k > 0$, the critical value of α_∞ is that corresponding to the maximum lift coefficient $C_{L_{max.}}$; for $k < 0$, h_0 can be calculated with $\alpha_\infty = 0$.

Controllability and Safety Against Stall

Eq. (2) shows how, for a given angle of attack of equilibrium, the required lift coefficient increases with the distance of the c.g. from the aerodynamic center of the wing (for flight test confirmation see reference 2). For every angle of attack of flight in equilibrium, there exists a limit forward c.g. location, which is that for which the required C_{L_t} is the maximum lift coefficient of the tail, $C_{L_{t_s}}$; and, with a given $C_{L_{t_s}}$, the maximum angle of attack at which the airplane can fly in equilibrium decreases as the c.g. moves forward.

It follows that the airplane is safe against stall if the tail lift coefficient required for equilibrium at the stalling angle of attack of the wing is $C_{L_{t_s}}$ when the c.g. is in its limit rearward location ($h = h_r$).

Conversely, the condition that the tail and the wing reach the stall simultaneously determines the limit rearward c.g. location. From Eq. (2), assuming that the vertical location of the c.g. be constant,

$$h_r = \frac{C_{L_{t_s}} l \frac{S_t}{S} + kC_{c_s} + (C_m)_W + \sum_i C_F j \alpha_{p_s}}{C_{L_{max.}} + C_{L_{t_s}} \frac{S_t}{S} + \sum_i C_F \alpha_{p_s}} \qquad (5)$$

where C_{c_s} is the chordwise force coefficient of the wing at stall and α_{p_s} is the inclination of the propeller axis when the airplane is at the stalling incidence.

If the vertical displacements of the c.g. are not negligible, Eq. (5) gives the locus (h_r, k) of the most rearward allowable c.g. locations for safety against stall.

Since the maximum incidence at which the airplane can fly decreases when the c.g. moves forward and since there is a lowest maximum lift coefficient C_{L_f} which the wing must be able to reach regardless of the c.g. location (e.g., the lift coefficient corresponding to the angle of attack of best rate of climb), it follows that the most forward allowable c.g. location, h_f, is determined by the condition that the airplane be able to fly at the incidence corresponding to C_{L_f} when $h = h_f$.

From Eq. (2),

$$h_f = \frac{C_{L_{t_s}} l \frac{S_t}{S} + kC_{c_s} + (C_m)_W + \sum_i C_F j \alpha_{p_f}}{C_{L_f} + C_{L_{t_s}} \frac{S_t}{S} + \sum_i C_F \alpha_{p_f}} \qquad (6)$$

where C_{c_f} is the chordwise force coefficient of the wing when $C_L = C_{L_f}$ and α_{p_f} is the corresponding inclination of the propeller axis.

If the vertical displacements of the c.g. are not negligible, Eq. (6) gives the locus (h_f, k) of the most forward allowable c.g. locations.

Power-Off and Power-On

The most rearward allowable c.g. location for static longitudinal stability, power-off, is:

$$(h_0)_0 = \frac{m_t \frac{S_t}{S} l + (2\alpha_\infty - \alpha_0)mk}{m + m_t \frac{S_t}{S}} \qquad (4')$$

If j is negative, or positive but smaller than $(h_0)_0$ (i.e., if the plane of the propeller is behind the most rearward allowable c.g. location for stability with power-off), then the introduction of the propeller terms as in Eq. (4) would yield $h_0 < (h_0)_0$. In this case, the critical condition is obviously that of power-off, and $(h_0)_0$ must be taken as the most rearward allowable c.g. location for stability.

If j is positive and greater than $(h_0)_0$, the critical condition is that of power-on, and the propeller terms have to be considered in the computation of the most rearward allowable c.g. location, as in Eq. (4).

If there are two or more propellers whose planes of rotation are at different distances from the aerodynamic center, then for the calculation of h_0 only the propeller terms for those propellers whose j is positive and greater than $(h_0)_0$ will have to be considered in Eq. (4).

The following is found by analogous considerations.

The most rearward allowable c.g. location for safety against stall, power-off, is:

$$(h_r)_0 = \frac{C_{L_{t_s}} l \frac{S_t}{S} + kC_{c_s} + (C_m)_W}{C_{L_{t_s}} \frac{S_t}{S} + C_{L_{max.}}} \qquad (5')$$

In the calculation of h_r, only the propeller terms pertaining to those propellers whose j is positive and greater than $(h_r)_0$ will have to be considered in Eq. (5).

The most forward allowable c.g. location for controllability, power-off, is:

$$(h_f)_0 = \frac{C_{L_{t_s}} l \frac{S_t}{S} + kC_{c_f} + (C_m)_W}{C_{L_{t_s}} \frac{S_t}{S} + C_{L_f}} \qquad (6')$$

If j is negative, or positive but smaller than $(h_f)_0$, then the introduction of the propeller terms as in Eq. (6) would yield $h_f < (h_f)_0$; therefore, the critical condition in this case is that of power-on.

If j is positive and greater than $(h_f)_0$, the condition of power-off is critical, and $(h_f)_0$ is the most forward allowable c.g. location.

If there are two or more propellers whose planes of rotation are at different distances from the aerodynamic center, only the propeller terms for those propellers whose j is negative, or positive and smaller than $(h_f)_0$, have to be taken into consideration in Eq. (6) for the determination of the most forward allowable c.g. location.

Of all possible conditions of power-on, the most critical is that of full throttle at the minimum speed. In such a condition J is a minimum; α_p, K and C_Q are great; and, therefore, F is at its maximum.

The Use of High-Lift Devices

From Eq. (2), it appears that if C_{L_t} were increased the airplane could be controlled to higher angles of attack when the c.g. is in a forward location.

Tail high-lift devices have been used for that purpose in both the Focke-Wulf and the Stefanutti canards. However, it is clear that the improved controllability is acquired at the expense of the safety against stall unless the most rearward allowable c.g. location is moved forward or unless a safety device is used to prevent the operation of the tail high-lift devices when the c.g. is aft of a certain location. Such a solution is evidently impractical.*

On the other hand, a tail high-lift device may be necessary when the wing is equipped with high-lift devices. The operation of the latter usually introduces a negative increment of $(C_m)_W$, whereby both h_f and h_r are reduced. Therefore, the condition of flaps neutral is critical for h_r, while the condition of flaps in operation is critical for h_f. In other words, the wing flaps have a tendency to reduce the range of the allowable c.g. locations. Such a reduction can be avoided by also equipping the tail with a high-lift device. But in such a case the high-lift devices of wing and tail must be interconnected: Eqs. (5) and (6) give the relation that must exist between the maximum lift coefficients of tail and wing, and therefore between the respective flap deflections, in order to keep h_f and h_r unchanged. Of course, C_{L_f} will change when the high-lift devices are in operation.

With Eq. (4), h_0 must be calculated for different conditions of operation of the high-lift devices, including the neutral condition. The greatest obtained value must be assumed as the most rearward allowable c.g. location for stability.

* The Focke-Wulf FW 19a did not have any safety device, and its range of allowable c.g. locations was reduced to 3.65 per cent of the mean chord—i.e., only about 3.6 in. (see reference 2).

Further Limitations

If $h_r < h_0$, then for $h_r < h < h_0$ the airplane would still be safe against stall but would be longitudinally unstable. Therefore, in such a case h_0 must be taken as the limit rearward c.g. location. However, that means a cut in the range of the c.g. locations. In addition, as the c.g. cannot be allowed to reach the position h_r, the airplane can never fly at the angle of attack of maximum lift. Hence, the condition $h_r < h_0$ is undesirable.

On the other hand, if $h_r > h_0$, the airplane may be too stable. As in the case of the conventional airplane, a condition near neutral stability when the c.g. is in the most rearward position is the most desirable. Such a condition is obviously achieved when

$$h_r = h_0 \tag{7}$$

If a certain range rc of c.g. locations is desired, also the equation

$$h_f - h_r = r \tag{8}$$

must be satisfied.

It is obvious that the conditions in Eqs. (7) and (8) are further limitations to the designer's freedom in proportioning the canard.

Sample Calculation

Data:	
Gross weight	4,000 lbs.
S	200 sq.ft.
Aspect ratio of wing	10
Aspect ratio of tail	4
m	0.085
m_t	0.070
c	5 ft.
$C_{L_{max}}$	1.6
C_{L_f}	1.3
$(C_m)_W$	0
α_{p_f}	10 deg. = 0.175 rad.
k	0
i	1
j	−1
D	8 ft.
J in the critical condition of power-on	0.6
$\left[\frac{C_Q}{J} - \frac{1}{2}\frac{dC_Q}{dJ}\right]$	0.025
K (from Fig. 2)	4.8
h_r	0.54
rc	6 in.

Determine tail length, tail area, aerodynamic characteristics of tail profile, angle of setting of the tail for different speeds and c.g. locations, minimum speed for different c.g. locations. From Eqs. (7) and (8), $h_r = h_0 = 0.54$ and $h_f = h_r + r = 0.64$.

Since j is negative, h_0 and h_r must be calculated for the condition of power-off. From Eq. (5'),

$$h_r = 0.54 = \frac{C_{L_{t_s}} l \frac{S_t}{200}}{C_{L_{t_s}} \frac{S_t}{200} + 1.6} \qquad \text{(a)}$$

The condition of power-on is critical for h_f. From Eq. (6),

$h_f = 0.64 =$

$$\frac{C_{L_{t_s}} l \frac{S_t}{200} - 4.8 \times \frac{64}{200} \times 0.025 \times 0.175}{1.3 + C_{L_{t_s}} \frac{S_t}{200} + 4.8 \times \frac{64}{200} \times 0.025 \times 0.175} \qquad \text{(b)}$$

Eq. (7) yields

$$\frac{\frac{0.070}{0.085} l(S_t/200)}{\frac{0.070}{0.085} \frac{S_t}{200} + 1} = \frac{C_{L_{t_s}} l(S_t/200)}{C_{L_{t_s}} \frac{S_t}{200} + 1.6}$$

whence $C_{L_{t_s}} = 1.32$—i.e., the tail must be such as to reach the stall when its lift coefficient is 1.32.

Then, from Eqs. (a) and (b), $S_t = 35$ sq.ft., $l = 4.27$, and $lc = 4.27 \times 5 = 21.4$ ft.

The tail lift coefficient C_{L_t} for equilibrium at different angles of attack and for different c.g. locations can be calculated from Eq. (2). The angle of setting of the tail is the difference between the angle of attack of the tail, which gives the tail lift coefficient C_{L_t}, and the angle of attack of the wing, which gives the corresponding wing lift coefficient C_L.

In Fig. 3 the values of C_{L_t} for equilibrium are plotted vs. the corresponding air speeds at sea level, instead of vs. the corresponding values of C_L. The weight of the airplane is assumed to be the same for all c.g. locations.

Fig. 3 also shows how the minimum speed increases as the c.g. moves forward.

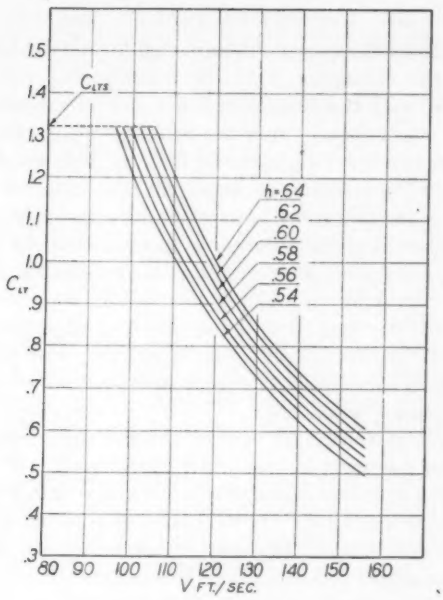

FIG. 3.

CONCLUSIONS

(1) The most rearward allowable c.g. location is determined by two conditions: static longitudinal stability and safety against stall. The condition of static longitudinal stability yields h_0 (Eq. 4); the condition of safety against stall yields h_r (Eq. 5). The greater of the two values must be taken as the most rearward allowable c.g. location.

(2) The maximum angle of attack at which a canard can fly in equilibrium decreases as the c.g. moves forward.

(3) The condition of safety against stall is attained at the expense of the maximum lift coefficient and, hence, of the minimum speed at which the airplane can fly in equilibrium. If $h_r \geq h_0$ so that h_r is the most rearward allowable c.g. location, the airplane can reach, but not exceed, the angle of attack of maximum lift coefficient when the c.g. is in the most rearward location. If $h_0 > h_r$ so that h_0 is the most rearward allowable c.g. location, then the canard can never reach the angle of attack of maximum lift coefficient, even when the c.g. is in the most rearward position.

(4) The lowest maximum lift coefficient C_{L_f} which the wing *must* be able to reach, regardless of the c.g. location, determines the most forward c.g. location (Eq. 6).

(5) Both h_f and h_r, and therefore also the range of the allowable c.g. locations if $h_r \geq h_0$, are linearly proportional to the tail length.

(6) The condition $h_r = h_0$ is desirable. This condition can be achieved by a proper selection of the profile or aspect ratio of the tail.

(7) Since the maximum lift coefficient that the wing can reach decreases as the c.g. moves forward, it is advisable that the useful load be distributed so that the c.g. is in the most rearward location in the full load condition and moves forward as the load decreases.

(8) High-lift devices on the tail, to be used only when the c.g. is in a forward location for the purpose of increasing C_{L_f}—i.e., the controllability—are dangerous and cannot be recommended.

(9) When the wing is equipped with high-lift devices, the use of high-lift devices on the tail, in conjunction with those operating on the wing, is recommended. The wing high-lift devices alone would usually decrease h_f and h_r, while the tail high-lift devices alone would increase both h_f and h_r. The operation of the wing and tail high-lift devices in conjunction with each other and under a single control can be done in such a way as to leave h_f and h_r unaffected. For every deflection of the wing high-lift devices, $C_{L_{max.}}$ and $(C_m)_W$ are known;

then, Eqs. (5) and (6) will yield $C_{L_{t_s}}$ (i.e., the corresponding deflection of the tail high-lift device) and C_{L_f} (i.e., the maximum wing lift coefficient that can be reached with the assumed deflection of the high-lift devices when the c.g. is in the most forward location).

(10) If the wing and tail high-lift devices are not operated in conjunction with each other under a single control and according to the rules described above, then the range of the allowable c.g. locations is reduced; h_f and h_r have to be calculated (Eqs. (5) and (6)) for all possible combinations of operation of the wing and tail high-lift devices; and the smallest of all values of h_f and the greatest of all values of h_r have to be taken as the most forward and the most rearward allowable c.g. locations, respectively.

(11) If the planes of rotation of the propellers are behind the aerodynamic center of the wing ($j < 0$), the condition of power-on is critical for h_f and that of power-off is critical for h_r. If the planes of rotation of all or some of the propellers are located before the aerodynamic center ($j > 0$), h_f and h_r must first be calculated for the condition of power-off [$(h_f)_0$ and $(h_r)_0$, Eqs. (5′) and (6′)]. Then, in the calculation of h_f (Eq. 6), only the propeller terms pertaining to those propellers whose j is less than the calculated $(h_f)_0$ must be considered; in the calculation of h_r (Eq. 5), only the propeller terms pertaining to those propellers whose j is greater than the calculated $(h_r)_0$ have to be considered. h_0 must be checked for the conditions of power-off and power-on in the same way as h_r.

(12) The angle of setting of the tail surface or the equivalent elevator deflection for equilibrium at a given angle of attack at which the wing lift coefficient is C_L can be calculated by means of Eq. (2) for any c.g. location. Eq. (2) yields C_{L_t} and therefore the angle of attack of the tail or the elevator deflection necessary to keep the airplane in equilibrium at the chosen angle of attack.

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Application of Maclaurin Series to Beams Under Simultaneous Transverse and Axial Loading

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INTRODUCTION

The method of determining the elastic line of columns and struts is, to start with, the fundamental equation $(d^2y/dx^2) = -M/EI$, and by two quadratures the deflection curve $y = f(x)$ is obtained. Most of the current methods carry this procedure through in some manner or other. While these methods are practical, it can be shown that in many cases other methods are shorter. As an illustration, application of the Maclaurin formula in solving for the deflection curve of beams was shown by Hetényi.[1] In this paper the writer extends the Maclaurin formula, which developed a function of real variable, say x, into a polynomial plus a remainder, to an infinite series known as the Maclaurin series, which expresses the function as a power series provided the remainder after n terms converges toward 0 as n become infinite.

Use of the Maclaurin series simplifies the solution of many problems and therefore should be of interest to engineers.

There is no approximation involved in this procedure since all terms are included, as will be evident from the expansion in the illustrative examples.

IN PROBLEMS RELATING to the bending of beams under simultaneous transverse and axial loading, this method begins with the relationship between curvature and moment in the familiar equation

$$(d^2y/dx^2) = -M/EI \tag{1}$$

where x and y are Cartesian coordinates, M equals the bending moment at the cross section through the point in question, I equals the moment of inertia of the cross section with respect to the neutral axis, and E equals the modulus of elasticity. The usual assumptions of small deflections, plane of cross section before bending remaining plane after bending, and homogeneity are implied.

If E and I are constant, and M, the bending moment, varies as some function of x and y, then Eq. (1) can be expressed symbolically as

$$F[(d^2y/dx^2), y, x] = 0 \tag{2}$$

a nonhomogeneous, second order, linear differential equation with constant coefficients, the solution of which can be found.

Instead of solving Eq. (2) by the standard procedures† found in texts on differential equations, the deflection curve $y = f(x)$ can be in a sense generated from the one equation—i.e., $f''(x) = (d^2y/dx^2) = -M/EI$.

It is recalled from the calculus that if $f(x)$ and all its derivatives exist and are continuous over an interval about $x = 0$ then $f(x)$ can be expanded in that interval in an infinite series of the form

$$y = f(x) = f(0) + f'(0)x + (f''(0)x^2/2!) + (f'''(0)x^3/3!) + \ldots + (f^n(0)x^n/n!) + \ldots \tag{3}$$

where n increases indefinitely and $f'(0)$, $f''(0)$, $f'''(0)$, $\ldots$ $f^n(0)$ are, respectively, the first, second, third, and nth derivative of $f(x)$ when $x = 0$. Maclaurin's infinite series, Eq. (3), represents $f(x)$ provided $(f^n(0)x^n/n!) \to 0$ as $n \to \infty$. The Maclaurin series expansion of $f(x)$ is known to be unique.‡ Thus given $f''(x) = (d^2y/dx^2) = -M/EI$, then by successive differentiation of the same with respect to x there results $f'''(x)$, $f^{IV}(x)$, $\ldots$ $f^n(x)$. Evaluating $f''(x)$, $f'''(x)$, $\ldots f^n(n)$ at $x = 0$ and then substituting in Eq. (3), a power series in x results, which in the cases discussed can be converted over into the circular and hyperbolic functions. $f(0)$ and $f'(0)$ are determined from the boundary conditions.

Eq. (3) can be written more compactly and this form will be found convenient although not necessary.**

$$y = f(0) + f'(0)x + \sum_{n=2,4,\ldots}^{n=\infty} [f^n(0)x^n/n!] + \sum_{n=3,5,\ldots}^{n=\infty} [f^n(0)x^n/n!] \tag{4}$$

Closely allied to the Maclaurin infinite series is Taylor's infinite series, which is used in some of the solutions of problems and is as follows:

$$y = f(\alpha) + f'(\alpha)(x-\alpha) + \sum_{n=2,4,\ldots}^{n=\infty} \times [f^n(\alpha)(x-\alpha)^n/n!] + \sum_{n=3,5,\ldots}^{n=\infty} [f^n(\alpha)(x-\alpha)^n/n!] \tag{5}$$

which states that the function $y = f(x)$ is evaluated in

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† By standard procedures is meant the more common methods such as reduction by differentiation to homogeneous form operator method, variation of parameters, etc.

‡ Derivation of Maclaurin and Taylor series can be found in Wilson's *Advanced Calculus*.

** The rearrangements of the terms of the power series are valid within the interval of convergence. See, for example, Wilson's *Advanced Calculus*, p. 442.

terms of the value of function and its derivatives at $x = \alpha$ in powers of $(x - \alpha)$.

The following expansion in the power series of functions encountered frequently in this paper are given here without proof.‡

$$(\cos kx - 1) = \sum_{n=2,4,\ldots}^{n=\infty} (-1)^{n/2} k^n x^n/n! \quad (6)$$

$$(\sin kx - kx) = \sum_{n=3,5,\ldots}^{n=\infty} (-1)^{(n-1)/2} k^n x^n/n! \quad (7)$$

$$(\cosh kx - 1) = \sum_{n=2,4,\ldots}^{n=\infty} k^n x^n/n! \quad (8)$$

$$(\sinh kx - kx) = \sum_{n=3,5,\ldots}^{n=\infty} k^n x^n/n! \quad (9)$$

Applications

In what follows, unless the contrary is specified, it is assumed that the moment of inertia I is constant, that the material is homogeneous, and that E is constant everywhere. It is assumed that the bending is uniplanar—i.e., that the bent centerline of the column lies entirely in one plane. The condition for this is that the plane of bending contain one of the principal axes of the cross section. The load P is applied at the center of area of the cross section and along the unstrained centerline. For convenience, let $k^2 = P/EI$ in the following illustrations.

Case 1. Cantilever Beam Under Axial Compression and Transverse End Load (Fig. 1)

At any point x

$$(d^2y/dx^2) = [Q(l - x)/EI] + [P(\delta - y)/EI] \quad (1.1)$$

From the end conditions: The deflection is zero when $x = 0$, the slope to the deflection curve is zero when $x = 0$, and when $x = l$ the deflection is equal to δ. Thus, the boundary conditions may be expressed as

$$f(0) = 0 \quad f'(0) = 0 \quad f(l) = \delta \quad (1.2)$$

Then by successive differentiation of Eq. (1.1) with respect to x and expansion about $x = 0$ and by the substitution of $k^2 = P/EI$, there results

$$f''(x) = \frac{Q(l-x)}{EI} + \frac{P(\delta - y)}{EI} \cdot \qquad f''(0) = \frac{Ql}{EI} + k^2\delta$$

$$f'''(x) = -\frac{Q}{EI} - \frac{Py'}{EI} \qquad f'''(0) = -\frac{Q}{EI}$$

$$f^{IV}(x) = -\frac{Py''}{EI} \qquad f^{IV}(0) = -\frac{Ql}{EI}k^2 - k^4\delta$$

$$f^{V}(x) = -\frac{Py'''}{EI} \qquad f^{V}(0) = +\frac{Q}{EI}k^2$$

$$\downarrow \qquad\qquad \downarrow$$

The nth term of $f^n(0)$ is

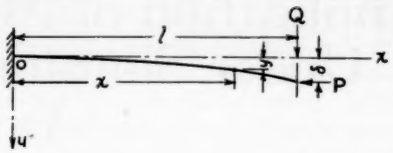

FIG. 1.

$$f^n(0) = -(-1)^{n/2}(Ql/EI)k^{n-2} - (-1)^{n/2}k^n\delta \quad \text{where } n = 2, 4, 6, \ldots$$

$$f^n(0) = (-1)^{(n-1)/2}(Q/EI)k^{n-3} \quad \text{where } n = 3, 5, 7, \ldots$$

From Eq. (1.2), $f(0) = f'(0) = 0$ and employing $f^n(0)$ from above, substitution in the Maclaurin infinite series, Eq. (4), gives

$$y = 0 + 0 - (Ql/EI)\sum_{n=2,4,\ldots}^{n=\infty}[(-1)^{n/2}k^{n-2}x^n/n!] - \delta \times \sum_{n=2,4,\ldots}^{n=\infty}[(-1)^{n/2}k^nx^n/n!] + (Q/EI)\sum_{n=3,5,\ldots}^{n=\infty}(-1)^{(n-1)/2}(k^{n-3}x^n/n!)$$

From (6) and (7) one can recognize the expansion of $(\cos kx - 1)$ and $(\sin kx - kx)$ after multiplying the first summation by k^2/k^2 and the third by k^3/k^3 and remembering that $k^2 = P/EI$, thus

$$y = -(Ql/P)(\cos kx - 1) - \delta(\cos kx - 1) + (Q/Pk)(\sin kx - kx) \quad (1.3)$$

In Eq. (1.3) the value of δ may be obtained by the use of the third boundary condition $y = \delta$ when $x = l$. Then

$$\delta = (Q/P)[(\tan kl/k) - l] \quad (1.4)$$

at $x = l$. The maximum bending moment is $M_{max.} = Ql + P\delta = (Q/k)\tan kl$ at $x = 0$ and the equation of the elastic line is obtained by substituting the value of δ from Eq. (1.4) into Eq. (1.3)

$$y = -(Q/Pk)\tan kl(\cos kx - 1) + (Q/Pk)\sin kx - (Qx/P)$$

Case 2

Case 2 is the same problem as in Case 1, except that the function is to be expanded about $x = l$. From Fig. 1 the slope is $f'(l)$ and is not zero, and $f(l) = \delta$. Then

$$f''(l) = 0$$
$$f'''(l) = -k^2f'(l) - (Q/EI)$$
$$f^{IV}(l) = 0$$
$$f^{V}(l) = +k^4f'(l) + (Q/EI)k^2$$
$$\downarrow$$

The nth term of $f^n(l)$ is

$$f^n(l) = 0 \quad \text{when } n = 2, 4, 6, \ldots$$
$$f^n(l) = (-1)^{(n-1)/2}f'(l)k^{n-1} + (-1)^{(n-1)/2}(Qk^{n-3}/EI) \quad \text{when } n = 3, 5, 7, \ldots$$

In this case $x = l$ has been arbitrarily chosen as a point of expansion; thus Taylor's infinite series (Eq. (5)) is used where α is replaced by l. Then

$$y = \delta + f'(l)(x - l) + f'(l) \times \sum_{n=3,5,\ldots}^{n=\infty} [(-1)^{(n-1)/2}k^{n-1}(x-l)^n/n!] + \frac{Q}{EI}\sum_{n=3,5,\ldots}^{n=\infty} [(-1^{(n-1)/2}k^{n-3}(x-l)^n/n!] \quad (2.1)$$

By inspection of Eq. (2.1), the odd n's suggest a sin kx expansion. Multiply the first summation by k/k and the second by k^3/k^3. From Eq. (7) and factoring, there results

$$y = \delta + (f'(l)/k)\sin k(x-l) + (Q/Pk) \times [\sin k(x-l) - k(x-l)] \quad (2.2)$$

when $x = 0$, $y = 0$; then

$$\delta = -(Q/Pk)(kl - \sin kl) + [f'(l)/k]\sin kl \quad (2.3)$$

when $x = 0$, $f'(0) = dy/dx = 0$; differentiating Eq. (2.2) and placing $x = 0$,

$$f'(l) = (Q/P)[(1 - \cos kl)/\cos kl] \quad (2.4)$$

Substituting $f'(l)$ from Eq. (2.4) into Eq. (2.3), δ becomes

$$\delta = (Q/P)[(\tan kl/k) - l] \quad (2.5)$$

This result is the same as Case 1, Eq. (1.4), thus demonstrating that the expansion may be taken about either end of the beam.

Case 3. Ideal Column. Position Fixed at Each End (Fig. 2)

Both ends are free in direction but fixed in position. This case, like many cases studied in elementary strength of materials, will demonstrate the ease and simplicity of the method.

Take the origin at 0, the midpoint of the length of the beam, then

$$d^2y/dx^2 = -Py/EI$$

The boundary conditions are

$$f(0) = \delta \quad f'(0) = 0 \quad f(l/2) = 0$$

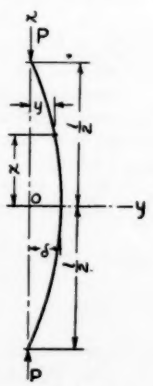

FIG. 2.

Thus

		Expansion About $(0, \delta)$
$f''(x)$	$= -Py/EI$	$f''(0) = -k^2\delta$
$f'''(x)$	$= -Py'/EI$	$f'''(0) = 0$
$f^{IV}(x)$	$= -Py''/EI$	$f^{IV}(0) = +k^4\delta$
$f^{V}(x)$	$= -Py'''/EI$	$f^{V}(0) = 0$
↓		↓
$f^n(0)$	$= 0$	when $n = 3, 5, 7, \ldots$
$f^n(0)$	$= (-1)^{n/2}k^n\delta$	when $n = 2, 4, 6, \ldots$

Then substituting in the Maclaurin series gives

$$y = \delta + \delta \sum_{n=2,3,4\ldots}^{n=\infty} (-1)^{n/2}k^n x^n/n! \quad (3.1)$$

or

$$y = \delta + \delta(\cos kx - 1) = \delta \cos kx \quad (3.2)$$

when $x = l/2$, $y = 0$, Eq. (3.2) becomes

$\cos(kl/2) = 0$ when $kl/2 = n\pi/2$ where $n = 1, 3, 5, \ldots$
or $k = \pi/l$
$k^2 = \pi^2/l^2 = P/EI$
or $P_{cr} = \pi^2EI/l^2$

Case 4. Bending of a Compressed Bar by a Couple (Fig. 3)

Both ends are fixed in position but free in direction. A couple M_B is applied at B. Take the origin at 0. Then

$$d^2y/dx^2 = [-(M_B/l)x - Py](1/EI)$$

Boundary conditions are

$$f(0) = 0 \quad f(l) = 0 \quad (4.1)$$

$$f''(x) = \left[-\frac{M_B}{l}x - Py\right]\frac{1}{EI} \qquad f''(0) = 0$$

$$f'''(x) = \left[-\frac{M_B}{l} - Py'\right]\frac{1}{EI}$$

$$f'''(0) = -\frac{M_B}{l}\frac{k^2}{P} - k^2f'(0)$$

$$f^{IV}(x) = -Py''/EI \qquad f^{IV}(0) = 0$$

$$f^{V}(x) = -\frac{Py'''}{EI} \qquad f^{V}(0) = \frac{M_B}{l}\frac{k^4}{P} + k^4f'(0)$$

↓ ↓

$f^n(0) = 0$ when $n = 2, 4, 6, \ldots$

$f^n(0) = (-1)^{(n-1)/2}(M_B/Pl)k^{n-1} + (-1)^{(n-1)/2}f'(0)k^{n-1}$ when $n = 3, 5, 7, \ldots$

Substituting in the Maclaurin series gives

$$y = 0 + f'(0)x + (M_B/Pl)\sum_{n=3,5,\ldots}^{n=\infty}(-1)^{(n-1)/2}(k^{n-1}x^n/n!) + f'(0)\sum_{n=3,5,\ldots}^{n=\infty}(-1)^{(n-1)/2}(k^{n-1}x^n/n!) \quad (4.2)$$

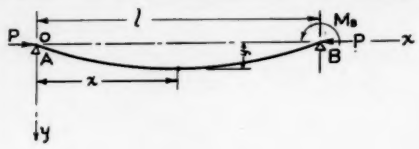

FIG. 3.

or

$$y = f'(0)(\sin kx/k) + (M_B/Plk)(\sin kx - kx) \quad (4.3)$$

when $x = l$, $y = 0$; then by substitution in Eq. (4.3) and solving for $f'(0)$

$$f'(0) = (M_B/P)[(k/\sin kl) - (1/l)] \quad (4.4)$$

Substituting $f'(0)$ from Eq. (4.4) into Eq. (4.3), then the equation of the elastic line is

$$y = (M_B/P)[(\sin kx/\sin kl) - (x/l)] \quad (4.5)$$

Case 5. Beam on End Supports Under Axial Compression and Triangularly Distributed Transverse Load (Fig. 4)

At any point x

$$d^2y/dx^2 = f''(x) = [-Py - (wlx/6) + (wx^3/6l)]1/EI$$

The boundary conditions are

$$f(0) = 0 \qquad f(l) = 0 \quad (6.1)$$

Expansion of $f''(x)$ about $x = 0$ can be shown to give the following $f^n(0)$.

$f^n(0) = 0$ when $n = 2, 4, 6, \ldots$

$f^n(0) = (-1)^{(n-1)/2}k^{n-1}f'(0) + (-1)^{(n-1)/2}k^{n-3}wl/6EI$ when $n = 3, 5, 7, \ldots$

$f^n(0) = (-1)^{(n-1)/2}k^{n-5}\,w/lEI$ when $n = 5, 7, 9, \ldots$

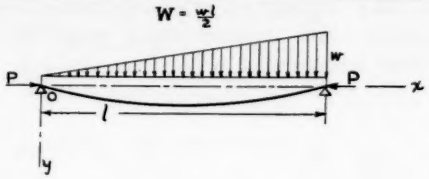

FIG. 4.

Thus, substitution in the Maclaurin series gives

$$y = f'(0)x + f'(0)\sum_{n=3,5,\ldots}^{n=\infty}\frac{(-1)^{(n-1)/2}k^{n-1}x^n}{n!} + \frac{wl}{6EI}\sum_{n=3,5,\ldots}^{n=\infty}\frac{(-1)^{(n-1)/2}k^{n-3}x^n}{n!} + \frac{w}{lEI}\sum_{n=5,7,\ldots}^{n=\infty}\frac{(-1)^{(n-1)/2}k^{n-5}x^n}{n!}$$

and

$$y = (f'(0)\sin kx/k) + (wl/6Pk)(\sin kx - kx) + (w/lEIk^5)[\sin kx - kx + (k^2x^3/6)] \quad (6.2)$$

when $x = l$, $y = 0$; then from Eq. (6.2)

$$f'(0) = w/P[(1/k\sin kl) - (1/lk^2) - (l/6)] \quad (6.3)$$

Substituting $f'(0)$ from Eq. (6.3) in Eq. (6.2) the equation of the elastic line becomes

$$y = w/P[(\sin kx/k^2\sin kl) + (x^3/6l) - (lx/6) - (x/k^2l)]$$

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Elliptic Tunnel-Wall Corrections on Drag and Stall

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SUMMARY

The tunnel-wall effects on drag and stall, given by the usual simple corrections, are not adequate when the span-diameter ratio is high or when the spanwise lift distribution varies considerably from elliptical. In order to determine more exact interference corrections, especially for elliptic throat tunnels of closed section, the upwash induced by the tunnel walls on the major axis, due first to the presence of lifting lines of different span lengths and then to the presence of typical wing distributions built up from these lifting lines to simulate actual wings, is computed for a number of wing distributions and spans. All the calculations are for wings whose aspect ratio is eight, and all are based on the constants of the Wright Brothers elliptic throat wind tunnel at the Massachusetts Institute of Technology.

The magnitude of these corrections is found to vary inversely with the taper ratio of the wing, the corrections being less for a highly tapered wing and more for a rectangular wing of the same span and aspect ratio. For those cases where the wing tips lie outside the foci of the throat section, the variation of the induced angle of attack along the span is no longer negligible at high lift coefficients. Its geometric effect is to cause an apparent washin angle toward the wing tip. The induced drag corrections have minimum values when the wing tips are at or near the foci of the tunnel for all usual lift distributions along the span. As the wing tips approach the tunnel wall, both the induced drag corrections and the angles of apparent washin or of apparent twist increase rapidly.

SYMBOLS

A = semimajor axis of tunnel cross section
B = semiminor axis of tunnel cross section
b = model wing span
c = focal distance of tunnel cross section
C_L = lift coefficient
ΔC_{D_i} = induced drag coefficient correction due to tunnel walls
L = lift (local force along span)
A.R. = aspect ratio
S_T = tunnel cross-sectional area
S_w = model wing area
T.R. = taper ratio = $\frac{\text{chord at wing root}}{\text{fictitious tip chord}}$
U = undisturbed stream velocity
W = induced upwash due to tunnel walls
y = distance from center of tunnel cross section along A
ξ = hyperbolic component in hyperbolic, ellipsoidal coordinate system
ξ' = ξ, corresponding to a given wing tip position (tip inside c)
η = ellipsoidal component in hyperbolic, ellipsoidal coordinate system
η_0 = constant η giving ellipse representing tunnel wall
a = η/η_0 for η between 0 and η_0
a' = a, corresponding to a given wing tip position (tip beyond c)
β = $y/(b/2)$
α = angle of attack
$\Delta\alpha$ = induced angle of attack due to tunnel walls
Γ = local circulation around wing
Γ_0 = circulation at wing root
K = Γ segment/Γ_0
δ_0 = upwash correction factor
δ_m = mean upwash correction factor for a given loading and span

TUNNEL CONSTANTS

$B/A = 0.732 \quad c = 0.68A$

$k = 1.584 \quad S_T = k\pi c^2$

THE CASE OF THE LIFTING LINE IN THE ELLIPTIC TUNNEL

THE FIRST STEP is to determine the tunnel-wall interference on a lifting line that lies symmetrically on the semimajor axis of the tunnel. Following customary procedure, the wall of the tunnel is replaced by a system of reflected vortices. Because the position of the vortices is not immediately apparent in the elliptic-hyperbolic coordinate system determined by the tunnel section, it is necessary to convert the system into a rectilinear system by means of the following transformation:[1]

$$Z = c \cos (\eta + i\xi) \tag{1}$$

In this rectilinear system it is possible to fix the posi-

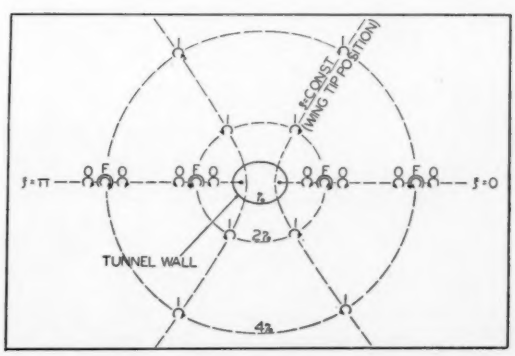

FIG. 1. Reflected vortex pattern elliptical wind tunnel. •—Focal points. I—Wing tips inside foci. F—Wing tips at foci. O—Wing tips outsiue foci.

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tion of the reflected vortices so that these, together with the vortices of the lifting line within the tunnel, produce the desired boundary conditions at the tunnel walls. These vortex positions can then be transferred back into the elliptic-hyperbolic system. This result is shown in Fig. 1 for three cases: tips of lifting line inside foci; at foci; and outside foci. This vortex pattern extends to infinity.

Using the vortex pattern a summation can be made to determine the wall-induced upwash at any point along the semimajor axis of the tunnel, either inside or outside the tips of the lifting line. The upwash, in the case where the tips of the lifting line are outside the foci of the tunnel, is:

$$W = \frac{\Gamma}{4\pi c} \sum_{n=2}^{n=\infty} (-1)^{\frac{n}{2}+1} \times \left[\frac{1}{\cosh (n-a)\eta_0 - \beta \cosh a\eta_0} + \frac{1}{\cosh (n-a)\eta_0 + \beta \cosh a\eta_0} + \frac{1}{\cosh (n+a)\eta_0 - \beta \cosh a\eta_0} + \frac{1}{\cosh (n+a)\eta_0 + \beta \cosh a\eta_0} \right] \quad (2)$$

where n is an even integer.

Similarly, for tips of the lifting line inside the tunnel foci the expression is:

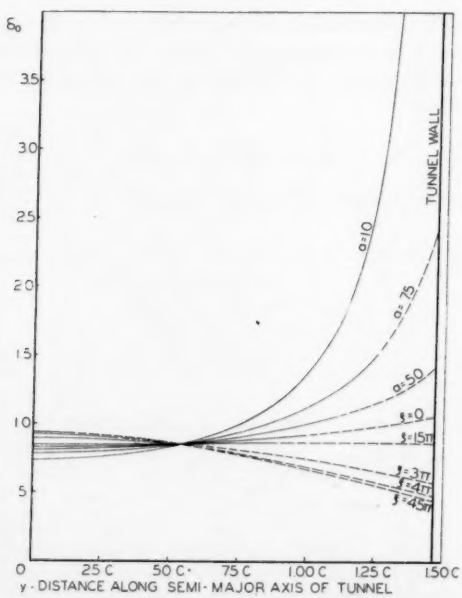

FIG. 2. Spanwise variation of upwash correction factor for lifting lines of varying lengths. Wright Brothers Wind Tunnel, M.I.T.

$$W = \frac{\Gamma \cos \xi}{2\pi c} \sum_{n=2}^{n=\infty} (-1)^{\frac{n}{2}+1} \times \left[\frac{\cosh n\eta_0 + \beta}{(\cosh n\eta_0 + \beta)^2 \cos^2 \xi + \sinh^2 n\eta_0 \sin^2 \xi} + \frac{\cosh n\eta_0 - \beta}{(\cosh n\eta_0 - \beta)^2 \cos^2 \xi + \sinh^2 n\eta_0 \sin^2 \xi} \right] \quad (3)$$

where n is an even integer.

These series converge rapidly and the first three terms yield accuracy within approximately 1 per cent. For convenience, these expressions can be rearranged, introducing a tunnel constant, $k = 1.584$.

Tips outside foci:

$$W = \frac{\Gamma c \cosh a\eta_0}{2k\pi c^2} \times \left\{ \frac{k}{2 \cosh a\eta_0} \sum_{n=2}^{n=\infty} (-1)^{\frac{n}{2}+1} \left[\begin{array}{c} \text{same bracketed} \\ \text{expression as Eq. (2)} \end{array} \right] \right\} \quad (4)$$

Tips inside foci:

$$W = \frac{\Gamma c \cos \xi}{2k\pi c^2} \times \left\{ k \sum_{n=2}^{n=\infty} (-1)^{\frac{n}{2}+1} \left[\begin{array}{c} \text{same bracketed} \\ \text{expression as Eq. (3)} \end{array} \right] \right\} \quad (5)$$

where $c \cosh a\eta_0$ and $c \cos \xi$ represent the semispans of the lifting lines for tip positions determined by a and ξ. δ_0 is the term included within the braces in Eqs. (4) and (5). The variation of δ_0 has been calculated for tip positions corresponding to:

$$\xi = 0.45\pi,\ 0.4\pi,\ 0.3\pi,\ 0.15\pi,\ 0$$

$$a = 0.5,\ 0.75,\ 1.00$$

For each lifting line tip position, δ_0 has been calculated for various values of β both on and beyond the span of the lifting line. The results are shown in Fig. 2.

Lift Distributions Approximated by Assemblies of Lifting Lines

The next step is to make use of these results to find the upwash variation along wings having normal lift distributions. The four lift distributions used[2, 3] are shown in Fig. 3a, which are distributions in the presence of the tunnel walls. Any given lift distribution can be approximated by a set of lifting lines having the correct length and strength. It must be remembered that it is necessary to take account out to the wing tip of the wall-induced upwash beyond the tips of all lifting lines that have spans shorter than that of the wing under consideration. In order to handle this operation, Eqs. (4) and (5) are modified further. From Eq. (5),

$$W = (\Gamma c \cos \xi / k 2\pi c^2)\delta_0 \quad (6)$$

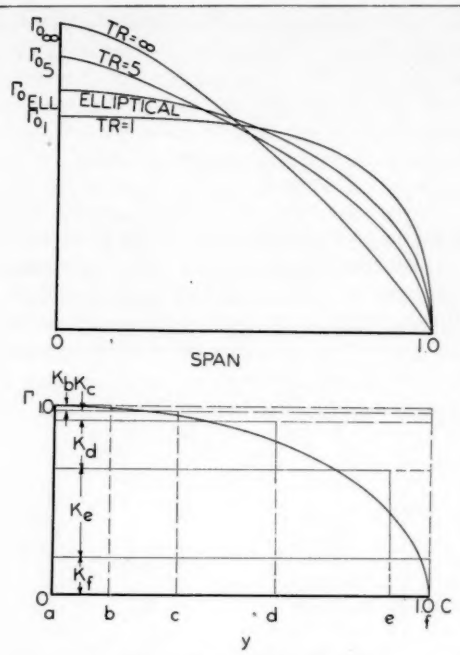

FIG. 3a. Lift distributions used.
FIG. 3b. Breakdown example. Elliptical distribution, tips at foci case.

This may be applied to a given lifting lines segment m of the total lift distribution to give the upwash at a given point p on the span of the wing:

$$\left(\frac{W}{U}\right)_{mp} = \frac{\Gamma_m c \cos \xi_m}{Uk2\pi c^2} \delta o_{mp} \tag{7}$$

A summation of the several segments yields

$$\left(\frac{W}{U}\right)_p = \frac{\Gamma_0}{Uk2\pi c^2}\left[\sum_m (K_m c \cos \xi_m \delta o_{mp})\right] \tag{8}$$

where $k_m\Gamma_0 = \Gamma_m$. For the elliptic distribution with tips inside the tunnel foci, it can be shown that

$$\Gamma_0/U = C_L S_W/\pi c \cos \xi' \tag{9}$$

By substituting in Eq. (8), remembering that $S_T = k\pi c^2$;

$$\left(\frac{W}{U}\right)_p = \frac{C_L S_W}{2\pi c \cos \xi' S_T}\left[\sum_m (K_m c \cos \xi_m \delta o_{mp})\right] \tag{10}$$

This is the upwash at a given point p on the span of a wing whose semispan is $c \cos \xi'$ and whose lift distribution is approximated by m lifting lines. Similarly, for the case where the wing tips lie outside the foci,

$$\left(\frac{W}{U}\right)_p = \frac{C_L S_W}{2\pi c \cosh a' \eta_0 S_T}\left[\sum_m (K_m c \cosh a_m \eta_0 \delta o_{mp})\right] \tag{11}$$

Then, Eqs. (10) and (11) can be generalized to form a single expression:

$$\left(\frac{W}{U}\right)_p = \frac{C_L S_W}{\pi b S_T}\left[\sum_m (K_m y_m \delta o_{mp})\right] \tag{12}$$

where b is the wing span and y_m is the semispan of the lifting line m.

The term $C_L S_W/\pi b S_T$ of Eq. (12) has been developed for the case of an elliptic lift distribution. However, by assuming a constant aspect ratio, the total lifts for all distributions for a given wing tip position are constant at a given lift coefficient. Hence, for other distributions, π is replaced by $\pi(\Gamma_{0_{ell.}}/\Gamma_{0_{other}})$.

Fig. 3*b* shows a typical breakdown of a lift distribution into its component lifting lines and illustrates how the factor K_m is determined. The lengths of the lifting ines are chosen to coincide with those used in the calculations of "The Case of the Lifting Line. . . " so that Fig. 2 can be used directly to find δo_{mp}. To determine the induced upwash variation along a given wing, the summation indicated by Eq. (12) is performed at several points p along the wing span. Fig. 4 shows the type of induced upwash variation occurring in cases with the wing tips inside, at and outside the tunnel foci.

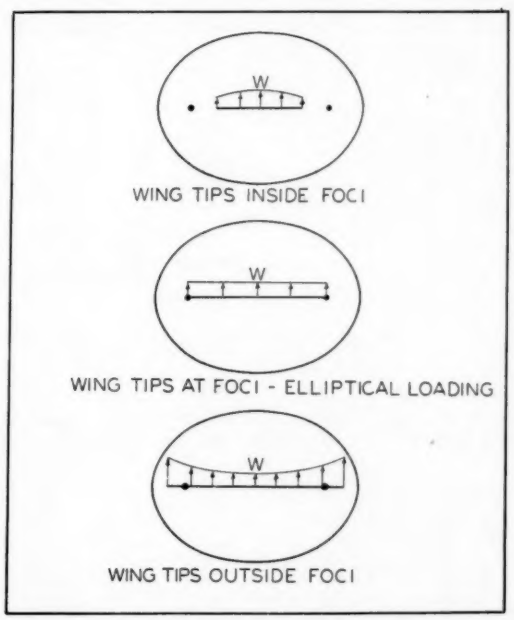

FIG. 4. Typical upwash distributions.

INDUCED UPWASH VARIATION

For each of the lift distributions used, wings having a constant aspect ratio of 8 were taken with their tips at the following five positions:

(1)	$\xi' = 0.3\pi$	$y' = 0.585C$	$b/2A = 0.40$
(2)	$\xi' = 0$	$y' = 1.000C$	$b/2A = 0.68$
(3)	$a' = 0.5$	$y' = 1.111C$	$b/2A = 0.76$
(4)	$a' = 0.75$	$y' = 1.255C$	$b/2A = 0.86$
(5)	$a' = 1.0$	$y' = 1.468C$	$b/2A = 1.00$

For each tip position the spanwise variation of the upwash corresponding to the four lift distributions was

plotted. Typical plots are shown in Figs. 5 and 6. As an illustrative example, the spanwise variation of wall-induced upwash has been reduced to the spanwise variation of induced angle of attack. This is shown in Fig. 7.

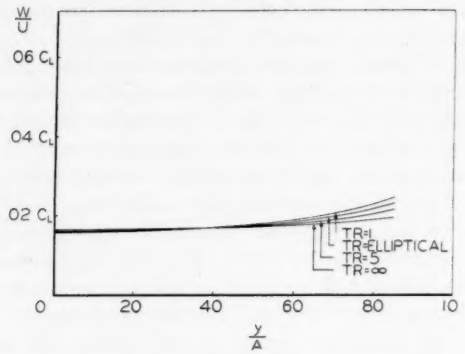

FIG. 5. Spanwise variation of upwash due to tunnel walls. Tip position: $a = 0.75;\ 1.255c;\ b/2A = 0.86$; A.R. = 8.

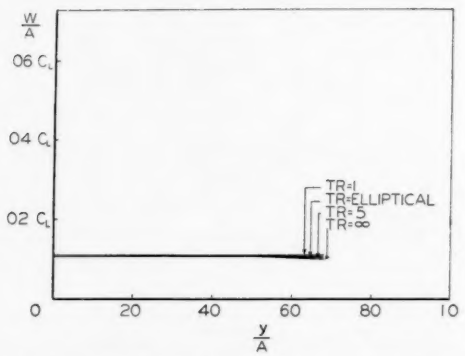

FIG. 6. Spanwise variation of upwash due to tunnel walls. Tip position: $\xi = 0;\ 1.000c;\ b/2A = 0.68$; A.R. = 8.

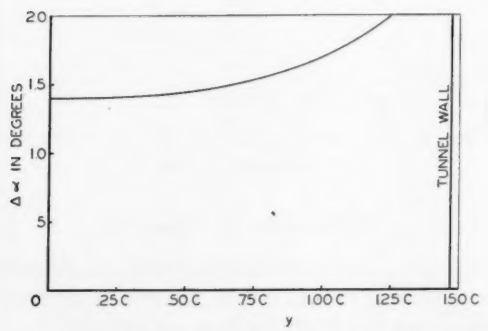

FIG. 7. Example of spanwise variation of induced angle of attack. Elliptical distribution. $C_L = 1.5$; $a = 0.75$; $1.255c$; $b/2A = 0.86$; A.R. = 8.

WALL-INDUCED DRAG AND ANGLE CORRECTIONS

In order to determine a mean upwash correction factor for the purpose of calculating induced drag correction and the mean wall-induced angle of attack, an average is taken of the upwash variations plotted. The development is as follows:

$$\Delta c_{Di} = C_L \left(\frac{W}{U}\right)_{av.} = \delta_M \frac{S_W}{S_T} C_L^2 \tag{13}$$

$$\delta_M = \frac{(W/U)_{av.}}{(S_W/S_T)C_L} \tag{14}$$

Thus, for each combination of wing tip position and lift distribution there is one value of δ_M. The variation of δ_M with wing tip position is plotted for each of the four lift distributions. This plot is presented in Fig. 8. For an infinitely small span the four cases merge and $\delta_M = {}^1/_8(\delta_0)_{\xi=\pi/2}$.

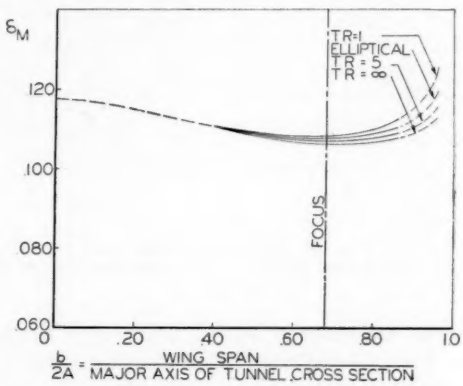

FIG. 8. Mean upwash correction factor. Variation with wing span. A.R. = 8. Wright Brothers Wind Tunnel, M.I.T.

An average induced angle of attack suitable for use as an angle correction due to the tunnel walls may then be taken as

$$\Delta\alpha_M = \delta_M(S_W/S_T)C_L \tag{15}$$

CONCLUSIONS

Before stating any conclusions it might be well to note again the restrictions that have been assumed in the numerical computations. The tunnel is of closed section. The eccentricity of the tunnel section enters immediately into consideration; hence, all results are limited to the Wright Brother Wind Tunnel or one with a similar throat section ($B/A = 0.732$). From the point where assemblies of lifting lines are used to approximate lift distributions of wings, the results are further limited to wings of A.R. = 8. The present calculations are in a form that facilitates application to other aspect ratios; a change in aspect ratio involves a constant percentage change in the values of induced upwash.

For the Wright Brothers Wind Tunnel, or any reasonably similar tunnel, the following generalities may be formed on the basis of the preceding calculations.

(1) At high lift coefficients, when the wing tip lies

outside the focal point, the variation of the induced angle of attack along the span is no longer negligible. This amounts to an apparent washin of angle which becomes severe as the wing tip approaches the wall. As a result, determinations of stall characteristics in the tunnel, when the wing span approaches the tunnel major diameter, are conservative—i.e., the wing tips will stall at higher angles of attack in free air.

(2) Other things being constant, tunnel-wall interference is less for lift distributions in which the lift is concentrated toward the centerline. That is, for untwisted wings, those with high taper ratio have tunnel-induced upwash of smaller magnitude than that of wings with low taper ratios (Figs. 5 and 6).

(3) Tunnel-wall interference is less for wings of high aspect ratio, other conditions being held constant. This fact may be seen from consideration of Eq. (12).

(4) By the use of a mean upwash correction factor, δ_M, as determined in this paper, the induced drag correction can be calculated more accurately than previously. Using δ_M in the determination of the induced angle of attack correction yields a result that is an average of the wall-induced angle of attack. This induced angle of attack correction is suitable for wind-tunnel work with the exception of stall pattern tests and is more accurate than that used formerly.

(5) For wings of normal lift distributions, the mean upwash correction factor, δ_M, is a minimum when the wing tips are approximately at the tunnel foci (Fig. 8).

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Note on Some Secondary Stresses in Thin-Walled Box Beams

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SUMMARY

An analysis is made of the secondary stresses that arise in flat-sheet-covered box beams in which the curvature of the cover sheets does not conform with the curvature assumed by the extreme fibers of the spars when the beam is loaded. Formulas are derived for rib loads and for secondary sheet bending stresses as function of the primary sheet stress, the rib spacing, the beam height and the cover sheet thickness. With regard to the secondary stresses, the flexibility of ribs and spars, which in principle can be taken into consideration, is neglected so that the values obtained for the stresses are conservative. It is found that secondary sheet bending is largely restricted to narrow edge zones adjacent to ribs and spars.

INTRODUCTION

THE PURPOSE of this article is to analyze the magnitudes of a group of secondary stresses in the cover sheets of thin-walled box beams. The effects that will be considered in what follows can be described qualitatively by saying that unless suitably stiffened the curvature of the cover sheets of box beams does not conform with the curvature assumed by the extreme fibers of the spars. The problem is analyzed here only for a cover sheet under tension, although a similar effect takes place in a cover sheet under compression before buckling sets in. The main results of the following developments are analytical expressions for the cover sheet bending stresses and for the rib loads due to straightening out of the cover sheets in between ribs and spars as functions of load intensity, rib spacing, beam height and cover sheet thickness.

While in many practical cases the calculated secondary stresses of this type will be small, it is shown by considering a numerical example that designs are possible for which the magnitudes of the secondary stresses are an appreciable fraction of the magnitude of the primary stress.

One of the conclusions from the analysis is the fact that the magnitude of the bending stresses obtained here may be such that experimental results concerning the effect of shear lag in cover sheets of box beams are appreciably distorted by the superposition of the straightening out effect.

FORMULATION OF THE PROBLEM

Consider a portion of a box beam between two adjacent ribs. The rib distance should be small enough that no appreciable change of bending moment takes place in the region considered (Fig. 1).

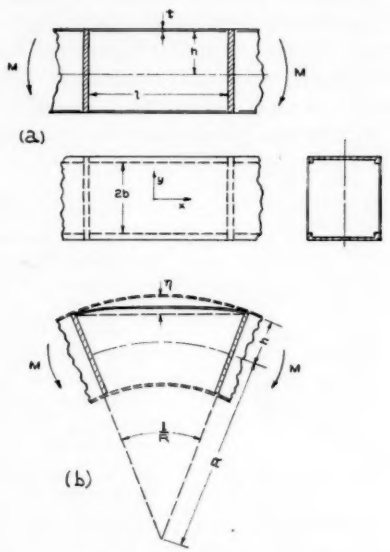

FIG. 1. Diagram of beam section in between two ribs: (a) undeformed; (b) deformed.

According to elementary beam theory, the neutral axis of the beam and all fibers parallel to it become curved. The length R of the radius of curvature is given by

$$1/R = M/EI \tag{1}$$

The spanwise stress in the cover sheets is given by

$$\sigma = (M/I)h \tag{2}$$

where h is the distance of the cover sheet plane from the neutral axis of the beam.

On the tension side this stress tends to straighten out the sheet and may succeed in doing so except for the portions of the sheet adjacent to the spars.

Since the straightened-out sheet fibers are shorter than the fibers attached to the slightly curved spars, a decrease toward the middle of the sheet of the direct spanwise stress σ will take place. It will be shown that in practical cases this decrease is exceedingly small and may safely be disregarded.

On the other hand, it will also be shown that the regions of the sheet which are not completely straightened out are extremely narrow and thus assume an

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appreciable amount of secondary curvature, with the consequence that there may occur appreciable bending stresses in a narrow edge zone adjacent to ribs and spars.

The Secondary Deflection of the Cover Sheet Due to Straightening-Out and the Resultant Reduction in Direct Stress

Introducing a system of coordinates x, y in the plane of the cover sheets, x spanwise and y transverse, the distance between curved and completely straightened sheet can be written as

$$w_0 = \eta[1 - 4(x^2/l^2)] \tag{3}$$

where η represents the maximum secondary deflection and l the distance between ribs.

Fig. 1b shows that η may be expressed in terms of h and R,

$$h + R - \eta = (h + R) \cos (l/2R) \tag{4}$$

Since

$$(l/2R) \ll 1 \quad h \ll R$$

the following formula can be written instead of Eq. (4)

$$\eta = l^2/8R \tag{5}$$

Expressing R in terms of σ by means of Eqs. (1) and (2), there follows for the maximum secondary deflection

$$\eta = (l^2/8h)(\sigma/E) \tag{6}$$

The reduction of the primary stress σ is given by the compressive stress $\Delta\sigma$, which reduces the length of the curved sheet to the length of the straightened-out sheet. The difference in length Δl of curved and straight sheet is given by

$$\Delta l = \int_{-l/2}^{l/2} \sqrt{1 + (dw_0/dx)^2}\, dx - l \tag{7}$$

the integral in Eq. (7) being the analytical expression for the arc length of the curve $w_0(x)$. Since $(dw_0/dx) \ll 1$ Eq. (7) can be simplified to

$$\Delta l = (1/2) \int_{-l/2}^{l/2} (dw_0/dx)^2 dx \tag{8}$$

In terms of this change of length the compressive stress $\Delta\sigma$ is given by

$$\Delta\sigma = E(\Delta l/l) \tag{9}$$

Introducing into Eq. (8) the function w_0 from Eq. (3) there follows first Δl in terms of η and l,

$$\Delta l = (8/3)(\eta^2/l) \tag{10}$$

and hence Eq. (9) becomes

$$\Delta\sigma = E\,(8/3)(\eta^2/l^2) \tag{11}$$

Introducing finally into Eq. (11) η from Eq. (6), there results for the relative reduction of stress due to straightening out of the sheet

$$\Delta\sigma/\sigma = (1/24)(l/h)^2(\sigma/E) \tag{12}$$

Since σ/E is not greater than $^1/_{100}$, it follows that for $\Delta\sigma$ to be more than $^1/_{100}$ of σ the distance between ribs has to be at least 50 times larger than the distance h between cover sheet plane and neutral axis of the beam, which condition will certainly never even be approximated.

Thus, Eq. (12) shows that there is no need to take into account the deformation of the beam cross section for the calculation of the direct cover sheet stresses except in cases outside the range of practical interest.

The Magnitude of the Rib Load Due to Straightening Out of the Sheet

Since the cover sheets of two adjacent bays, when straightened out, are inclined to each other, a component of the direct sheet stresses will be acting in the plane of the ribs. The magnitude of this component actually depends on the flexibility of the ribs. Here a conservative estimate of this magnitude is obtained on the basis of neglecting the flexibility of the ribs.

From Fig. 1 it is seen that the angle of inclination between two adjacent straightened-out sheets is of magnitude l/R. The resultant σ_R of the two inclined stresses σ acts in the plane of the rib and is of magnitude

$$\sigma_R = (l/R)\sigma \tag{13}$$

With R taken from Eqs. (1) and (2), σ_R can be written in the form

$$\sigma_R = (l/h)(\sigma^2/E) \tag{14}$$

To obtain the load p_R acting on a unit length of the rib, σ_R has to be multiplied by the cover sheet thickness t; hence,

$$p_R = \sigma t(l/h)(\sigma/E) \tag{15}$$

This load intensity p_R will be acting over that portion of the width where complete straightening out takes place. It will be shown that this is the case all along the rib except in a narrow edge zone adjacent to the spars, so that it is only slightly more conservative to assume that the load intensity p_R is uniform all along the rib. Eq. (15) indicates that the rib load intensity due to the straightening-out effect is directly proportional to the ratio of rib distance and cover sheet-neutral axis distance and proportional to the square of the direct sheet stress σ. A numerical example will be considered later.

Determination of the Sheet Bending Stresses

The determination of the bending stresses may be effected by considering the sheet as a thin plate having a slight initial deflection of magnitude $w_0(x)$. The plate is acted upon by spanwise edge normal forces of magnitude σt per unit of edge length (Fig. 2). For this kind of loading the differential equation for the final deflection w_1 of the plate is known to be of the form

$$\nabla^2\nabla^2 w_1 = (\sigma t/D)(\partial^2 w_1/\partial x^2) \tag{16}$$

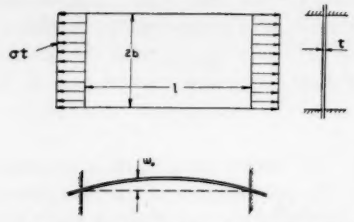

FIG. 2. Diagram for plate problem.

where

$$\nabla^2 \equiv (\partial^2/\partial x^2) + (\partial^2/\partial y^2) \qquad D = Et^3/12(1 - \nu^2) \quad (17)$$

As boundary conditions are prescribed, along the ribs

$$x = \pm l/2 \quad w_1 = 0 \quad \partial w_1/\partial x = \partial w_0/\partial x \quad (18)$$

and along the longitudinal supports, the distance between which may be designated by $2b$,

$$y = \pm b \quad w_1 = w_0 \quad \partial w_1/\partial y = 0 \quad (19)$$

To make the boundary conditions of the problem homogeneous, introduce the difference between the initial and final deflection

$$w = w_0 - w_1 \quad (20)$$

into the system of Eqs. (16) to (19). There follows for w the following system of equations:

$$\nabla^2\nabla^2 w = \frac{\sigma t}{D}\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w_0}{\partial x^2}\right) \quad (21)$$

$$x = \pm l/2 \quad w = 0 \quad \partial w/\partial x = 0 \quad (22)$$

$$y = \pm b \quad w = 0 \quad \partial w/\partial y = 0 \quad (23)$$

It should be noted that the boundary conditions of the problem are formulated as if the support offered to the sheet by ribs and spars were of an unyielding nature. This is certainly an idealizing assumption and the existing flexibility of the supports will reduce the actual stresses below those calculated on the basis of rigid supports. There is then obtained a conservative estimate only of the magnitude of the effect under consideration, with the possibility left open to improve the accuracy of the results by modifying appropriately the conditions of support of the plate.

The exact solution of the clamped-edge plate problem, Eqs. (21) to (23), offers considerable difficulty.

It should be observed, however, that the sheet bending along the longitudinal supports is dependent mainly on the conditions of support along the longitudinals and on the conditions along the ribs only to the extent that vanishing of the deflection there is essential, whereas the sheet bending along the ribs is due to the slope and deflection restraint offered by the ribs, while the support offered by the longitudinals is restricted to longitudinal edge zones (as will be shown). Therefore, the following simplification can be introduced. The problem is subdivided into two modified problems, one giving the stresses adjacent to the ribs (problem 1) and one giving the stresses adjacent to the spars (problem 2).

In problem 1 the support offered by the spars is neglected, and with that the transverse variation of the deflection, so that the sheet is assumed to behave like an initially slightly curved beam acted upon by axial tension.

In problem 2 the conditions of restraint at the ribs are assumed to be those of moment-free support.

Determination of the Stresses Adjacent to the Ribs

Considering problem 1, the differential equation and boundary conditions reduce to

$$\frac{d^4 w}{dx^4} - \frac{\sigma t}{D}\frac{d^2 w}{dx^2} = \frac{\sigma t}{D}\frac{8\eta}{l^2} \quad (24)$$

$$x = \pm l/2 \quad w = 0 \quad dw/dx = 0 \quad (25)$$

where the value of w_0 has been taken from Eq. (3). Putting as abbreviations

$$\frac{\sigma t}{D} = \frac{\sigma}{E}\frac{12(1 - \nu^2)}{t^2} = k^2 \quad (26)$$

$$k\frac{l}{2} = \sqrt{3(1 - \nu^2)}\sqrt{\frac{\sigma}{E}}\frac{l}{t} = \mu \quad (26a)$$

the solution of Eqs. (24) and (25) becomes

$$w = \eta\left[\left(1 - 4\frac{x^2}{l^2}\right) - 2\frac{\cosh \mu - \cosh kx}{\mu \sinh \mu}\right] \quad (27)$$

Thus the final sheet elevation w_1 is given by the expression

$$w_1 = w_0 - w = 2\eta\frac{\cosh \mu - \cosh kx}{\mu \sinh \mu} \quad (28)$$

For the secondary bending stress in the extreme sheet fibers, which is due to the relative deflection w, there follows from Eq. (27)

$$\sigma_x = \pm\frac{E}{1 - \nu^2}\frac{t}{2}\frac{d^2 w}{dx^2} = \pm$$

$$\frac{E}{1 - \nu^2}\frac{t}{2}\eta\left(-\frac{8}{l^2} + \frac{2k^2 \cosh kx}{\mu \sinh \mu}\right)$$

which may be written in the form

$$\sigma_x = \pm\frac{E}{1 - \nu^2}\frac{4t\eta}{l^2}\left(\frac{\mu \cosh kx}{\sinh \mu} - 1\right) \quad (29)$$

Inspection of Eq. (29) reveals that σ_x decreases rapidly from the edges $x = \pm l/2$ when $\mu \gg 1$, which is also found to be the condition for occurrence of appreciable bending stress. Writing then

$$\cosh kx/\sinh \mu \approx e^{-\mu\left(1 - \frac{x}{l/2}\right)} \quad (30)$$

there follows from Eq. (29)

$$\sigma_x = \pm\frac{E}{1 - \nu^2}\frac{4t\eta}{l^2}\mu e^{-\mu\left(1 - \frac{x}{l/2}\right)} \quad (31)$$

Eq. (31) shows that the maximum stress occurs directly adjacent to the ribs and decreases rapidly so that at a distance

$$\delta_r \approx 1.2(l/\mu) \tag{32}$$

it amounts to only 10 per cent of the edge stress. Introducing the value of μ from Eq. (26) and the value of η from Eq. (6), there follows for the maximum bending stress adjacent to the ribs and for the width of the edge zone

$$\frac{\sigma_r}{\sigma} = \pm\sqrt{\frac{0.75}{1-\nu^2}}\,\frac{l}{h}\sqrt{\frac{\sigma}{E}} \tag{33}$$

$$\delta_r \approx 0.8t\sqrt{\frac{E}{\sigma}} \tag{34}$$

Eq. (33) indicates that σ_r increases more rapidly than σ and that it is directly proportional to ratio of rib distance l and distance h from sheet to neutral axis, which ratio occurred previously in Eq. (15) for the rib loads p_R. Eq. (24) shows that the width of the edge zone is proportional to the sheet thickness and decreases with increasing primary stress σ.

It may be said that the value of the rib load intensity p_R as given by Eq. (15) can also be obtained as the difference in vertical shear between two adjacent ribs—that is, from the expression

$$^1/_2 p_R = -D(d^3w/dx^3)_{x=l/2}$$

Determination of the Stresses Adjacent to the Spars

To obtain values of the bending stresses along the spars, problem 2 is to be solved, which consists of the differential equation, Eq. (21), the boundary conditions, Eqs. (23), and the modified boundary conditions

$$x = \pm l/2 \quad w = 0 \quad M_x = 0 \tag{35}$$

A suitable solution of the differential equation is obtained by assuming

$$w = \cos \alpha x\, f(y) + \eta\left(1 - 4\frac{x^2}{l^2}\right) \qquad \alpha = \pi/l \tag{36}$$

This satisfies the displacement condition in Eqs. (35) exactly and the moment condition very nearly, so that these conditions are considered to be taken care of.

Introducing Eq. (36) into the differential Eq. (21), there follows for the function $f(y)$,

$$\frac{d^4f}{dy^4} - 2\alpha^2\frac{d^2f}{dy^2} + \left(\alpha^4 + \frac{\sigma t}{D}\alpha^2\right)f = 0 \tag{37}$$

which with the notations of Eqs. (26) may be written

$$\frac{d^4f}{dy^4} - 2\alpha^2\frac{d^2f}{dy^2} + \alpha^4\left(1 + \frac{4}{\pi^2}\mu^2\right)f = 0 \tag{38}$$

Since, as before, $\mu \gg 1$ within the range of interest, the solution of Eq. (38) is very nearly the same as the solution of the following simpler equation,

$$(d^4f/dy^4) + (4/\pi^2)\mu^2\alpha^4 f = 0 \tag{39}$$

Putting

$$\alpha^4\mu^2/\pi^2 = \gamma^4 \tag{40}$$

the solution of Eq. (39) is of the form

$$f = e^{-\gamma(b-y)}(C_1 \sin \gamma y + C_2 \cos \gamma y) + e^{\gamma(b-y)}(C_3 \sin \gamma y + C_4 \cos \gamma y) \tag{41}$$

This introduced into Eq. (36) represents the solution for w in which the constants of integration C_n are to be determined by means of the boundary conditions Eqs. (23).

The form of the function $f(y)$ indicates that its effect will be restricted to zones in the immediate neighborhood of the edges $y = \pm b$, and it is possible to simplify the solution by considering separately the effects along the two edges. Considering the edge $y = b$, $C_3 = C_4 = 0$ may be put so that

$$w \approx \cos \alpha x\, e^{-\gamma(b-y)}(C_1 \sin \gamma y + C_2 \cos \gamma y) + \eta\,[1 - 4(x^2/l^2)] \tag{42}$$

The boundary conditions, Eqs. (23), are sufficiently nearly satisfied in view of the similarity of the curves represented by $\cos \alpha x$ and by $[1 - 4(x^2/l^2)]$ by prescribing $w(0, b) = 0$ instead of $w(x, b) = 0$ so that the equations for the constants of integration become

$$w(0, b) = 0 \quad C_1 \sin \gamma b + C_2 \cos \gamma b = -\eta \tag{43}$$

$$(\partial w/\partial y)_{y=b} = 0 \quad C_1(\sin \gamma b + \cos \gamma b) + C_2(\cos \gamma b - \sin \gamma b) = 0 \tag{44}$$

Solving for C_1 and C_2 and substituting in Eq. (42),

$$w = \eta\left\{\cos \alpha x e^{-\gamma(b-y)}[\cos \gamma(b-y) + \sin \gamma(b-y)] - \left(1 - 4\frac{x^2}{l^2}\right)\right\} \tag{45}$$

For the bending stress normal to the spars in the extreme sheet fibers there follows, in accordance with plate theory,

$$\sigma_y = \pm\frac{E}{1-\nu^2}\frac{t}{2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \approx \pm\frac{E}{1-\nu^2}\frac{t}{2}\frac{\partial^2 w}{\partial y^2} \tag{46}$$

the term proportional to ν being negligible because γ is large compared with α. Hence,

$$\sigma_y = \mp\frac{E}{1-\nu^2}\frac{t}{2}\eta \times \{2\gamma^2 \cos \alpha x\, e^{-\gamma(b-y)}[\cos \gamma(b-y) - \sin \gamma(b-y)]\}$$

Taking γ of Eq. (40) which has the value

$$\gamma = \alpha\sqrt{\frac{\mu}{\pi}} = \frac{\pi}{l}\sqrt{\frac{\sqrt{3(1-\nu^2)}}{\pi}}\sqrt{\frac{\sigma}{E}}\frac{l}{t} \tag{47}$$

and η from Eq. (6) so that

$$t\eta\gamma^2 = t\frac{l^2}{8h}\frac{\sigma}{E}\frac{\pi^2}{l^2}\frac{\sqrt{3(1-\nu^2)}}{\pi}\sqrt{\frac{\sigma}{E}}\frac{l}{t} = \frac{\pi\sqrt{3(1-\nu^2)}}{8}\sqrt{\left(\frac{\sigma}{E}\right)^3}\frac{l}{h} \quad (48)$$

there follows, if one also puts

$$\frac{\pi\sqrt{3}}{8\sqrt{1-\nu^2}} = 0.71 \qquad \sqrt[4]{3\pi^2(1-\nu^2)} = 2.28$$

$$\frac{\sigma_y}{\sigma} = 0.71\frac{l}{h}\sqrt{\frac{\sigma}{E}}\cos\frac{\pi x}{l}e^{2.228\sqrt[4]{\frac{\sigma}{E}}\frac{b-y}{\sqrt{lt}}}\times$$

$$\left[\cos\left(2.28\sqrt[4]{\frac{\sigma}{E}}\frac{b-y}{\sqrt{lt}}\right) - \sin\left(2.28\sqrt[4]{\frac{\sigma}{E}}\frac{b-y}{\sqrt{lt}}\right)\right] \quad (49)$$

The stress σ_y takes on its maximum value σ_s adjacent to the spars and halfway in between the ribs. From Eq. (49) it is seen that

$$\sigma_s = 0.71\sigma(l/h)\sqrt{\sigma/E} \quad (50)$$

Comparison of σ_s with σ_r of Eq. (33) shows identical analytic expressions except for the value of the numerical coefficient, which in σ_r is about 20 per cent larger than in σ_s.

Eq. (49) indicates that the σ_y curve has the character of a damped oscillation, however, so rapidly damped that the values of σ_y beyond the first half wave are small fractions of σ_s.

The width δ_s of the edge zone can here be defined as the distance from the spar to where σ_y becomes zero for the first time. From Eq. (49) it follows that this is the case when

$$2.28\sqrt[4]{\sigma/E}\,(b-y)/\sqrt{lt} = \pi/4$$

whence

$$\delta_s = 0.35\sqrt{lt}\sqrt[4]{E/\sigma} \quad (51)$$

Comparison of Eq. (51) with Eq. (34) shows that there are different analytic laws for δ_s and for δ_r but that in both cases the occurrence of the sheet thickness t as a factor indicates the narrowness of the edge zone. It is because of this narrowness that the secondary sheet curvatures and, with that, the sheet bending stresses in the edge zone are rather larger than would otherwise be the case.

With the help of the solution obtained it is also possible to determine the forces between sheet and spars due to the straightening out of the sheet. These forces are the vertical reactions corresponding to the plate deflection as given by Eq. (45). According to plate theory the load intensity acting on the spars is here

$$p_s = -D\left(\frac{\partial^2 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial y \partial x^2}\right)_{y=b} \quad (52)$$

Because γ is large compared with α the second term on the right side of Eq. (52) may be neglected. Introducing w from Eq. (45) into the simplified Eq. (52) there follows

$$p_s = -D4\gamma^3\eta\cos\alpha x \quad (53)$$

and introducing the values of η, D, and γ from Eqs. (6), (17), and (47) there results, finally,

$$p_s = -0.54\sigma t\frac{\sqrt{lt}}{h}\left(\frac{\sigma}{E}\right)^{3/4}\cos\pi\frac{x}{l} \quad (54)$$

Comparing this result for the spar load intensity p_s with Eq. (15) for the rib load intensity p_R, it is seen that, in contrast with the expressions for the sheet bending stress, p_s and p_R are given by different analytic expressions. The important difference between p_s and p_R is that a factor l in Eq. (15) is replaced by a factor $\sqrt{lt}$ in Eq. (54) so that p_s is of a smaller order of magnitude than p_R.

A Numerical Example

To obtain an impression of the magnitude of the secondary stresses, a beam section of the following dimensions may be taken: rib distance $l = 10$ in.; distance of cover sheet from neutral axis $h = 5$ in.; sheet thickness $t = 0.1$ in.; cover sheet stress $\sigma = 10^5$ lbs. in.$^{-2}$; modulus of elasticity $E = 10^7$ lbs. in.$^{-2}$

There follows:

(1) From Eq. (15) for the rib load per inch,

$$p_R = 200 \text{ lbs. in.}^{-1}$$

(2) From Eq. (33) for the secondary sheet bending stress adjacent to the rib,

$$\sigma_r = 0.18\sigma = 18{,}000 \text{ lbs. in.}^{-2}$$

(3) From Eq. (48) for the secondary sheet bending stress adjacent to the spar,

$$\sigma_s = 0.14\sigma = 14{,}000 \text{ lbs. in.}^{-2}$$

(4) From Eq. (34) for the width of the edge zone at the rib,

$$\delta_r = 0.8t = 0.8 \text{ in.}$$

(5) From Eq. (49) for the width of the edge zone at the spar,

$$\delta_s = 1.1\sqrt{10t} = 1.1 \text{ in.}$$

(6) From Eq. (54) for the spar load intensity,

$$p_s = -35\cos(\pi x/l) \text{ lbs. in.}^{-1}$$

Finally, one may calculate the value of the maximum initial deflection η from Eq. (6),

$$\eta = 0.025 \text{ in.} = 0.25t$$

This result for η confirms the supposition that it is appropriate to apply the small deflection theory of plates to the problem at hand.

Oxygen Requirements at High Altitudes

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SUMMARY

Theoretic equations are developed to determine what concentrations of oxygen must be breathed at any altitude to maintain the "oxygen equivalent" of air at given altitudes—i.e., the same alveolar oxygen pressure. The partial pressures of water vapor and carbon dioxide are taken into account, as well as the variation of the pressure of carbon dioxide with the alveolar oxygen pressure.

Six curves are given on a chart showing the oxygen concentrations required at any altitude to maintain the oxygen equivalent of air at sea level and at altitudes of 5,000, 10,000, 15,000, 20,000, and 25,000 ft. Altitudes of 33,800, 40,000, and 46,000 ft. with pure oxygen are equivalent to air at sea level, 10,000 and 20,000 ft., respectively.

It is shown that these equivalents are nearly independent of the assumptions as to the state of physiologic adaptation and should be applicable to all individuals.

IT IS GENERALLY RECOGNIZED that most of the physiologic dangers of high altitude are due to lack of oxygen and may be largely avoided by the use of supplemental oxygen to maintain normal oxygen pressure in the lungs. The main exception is the danger from aeroembolism, which arises from gases coming out of solution in the body. Both qualitative and quantitative studies have been made on aeroembolism,[1, 2] and a great deal of experimentation has been done on the qualitative aspects of anoxia.[1, 3] However, data on the quantitative aspects of prevention of anoxia are still rather scarce.

The important question—What percentages of oxygen are required in the inspired air at various altitudes to maintain an alveolar oxygen pressure equivalent to that obtained with a given oxygen percentage at a given altitude?—has not been experimentally investigated in detail. It is the purpose of this note to present the derivation of a formula and a chart based on it for theoretic determination of such respiratory equivalents.

RESPIRATORY EQUATIONS

The respiratory equations must be deduced from considerations of the effect of water vapor and CO_2 on the partial pressure of oxygen in the inspired air.

For convenience, the symbols used in the development will be defined here.

V = ventilation, or volume rate of breathing, including atmospheric air and added oxygen, if any, in liters per minute, at ambient pressure and 0°C.
B = ambient atmospheric pressure, mm. Hg
POA = partial pressure of oxygen in dry inspired air, mm. Hg
$POL_{max.}$ = maximum oxygen pressure in upper part of lungs, mm. Hg
POL = partial pressure of oxygen in the lung alveoli, mm. Hg
PCL = partial pressure of CO_2 in the lung alveoli, mm. Hg
C = POA/B = fractional concentration of oxygen
PO = decrease in oxygen pressure in lungs due to absorption, mm. Hg

The maximum oxygen pressure in air moistened by passage through the respiratory tract is

$$POL_{max.} = POA[(B - 47)/B]$$

At sea level

$$POL_{max.} = [(21/100) \times 760] \times [(760 - 47)/760] = 150$$

It may be noted that this is independent of the humidity of the inspired air, moisture being supplied by evaporation as required so that the air is finally saturated at nearly body temperature.

The effect of CO_2 must also be considered explicitly. The inspired air is diluted with water vapor, of pressure 47 mm. Hg, largely while being drawn through the nasal passages; but, before reaching the absorbing tissues of the alveoli, it is further diluted by CO_2, of a pressure normally about 40 mm. Hg. Thus, the maximum oxygen pressure before absorption would be

$$POL = POA[(B - 47 - PCL)/B] \qquad (1)$$

which, at sea level, equals 140 mm. Hg.

If only oxygen, CO_2 and water vapor are present, POL in Eq. (1) is also the effective oxygen pressure at the absorbing alveolar walls.

If nitrogen or other diluent is present, local absorption of oxygen will effectively reduce the partial pressure of oxygen. Analysis of the last part of the air in a forced exhalation (alveolar air) indicates that the oxygen pressure in the alveoli averages about 100 mm. Hg for normal men breathing air at sea level (Fig. 1). This difference would indicate a pressure reduction due to absorption of about 40 mm. Hg; but, for the moment, call this decrease merely ΔPO.

To generalize these equations for all altitudes, for all oxygen concentrations, and for all persons, it is necessary to use the values for alveolar CO_2 pressure and of the oxygen consumption, characteristic of the person, and allow for their possible variation.

The equation

$$POL = POA[(B - 47 - PCL)/B] - \Delta PO \qquad (2)$$

is perfectly general but useless until values for PCL and ΔPO are given. PCL may vary from 20 to 40

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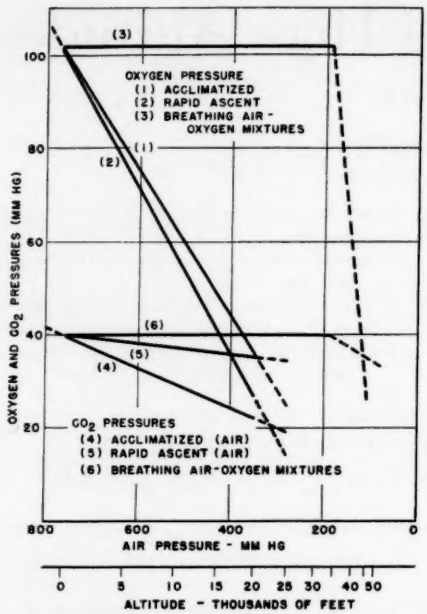

FIG. 1. Alveolar gas pressures for persons at various oxygen pressures or altitudes. Curves (1) and (4) are for men acclimatized at the indicated altitudes; (2) and (5) are for men taken from sea level to the indicated altitudes; (3) and (6) are for persons breathing air-oxygen mixtures to maintain normal O_2 and CO_2 pressures in the lungs.

mm. Hg, depending on the altitude to which the person is accustomed. Since the effects of high altitudes on persons who live near sea level are of the most concern, PCL may usually be taken as equal to 40. However, one effect of anoxemia is to increase the breathing volume, which washes the CO_2 out more rapidly and lowers its partial pressure, so that under conditions of plain anoxia, the approximation

$$PCL = 40(Vo/V) \tag{3}$$

might be used, where Vo is the normal, and V is the increased, rate of ventilation. At sea level ΔPO is about 40 mm. Hg when air is breathed and, as has been mentioned above, will be zero when pure oxygen is breathed. As a possible law of variation with concentration, let

$$\Delta PO = 50[1 - (POA/B)] \tag{4}$$

which satisfies the two conditions given with a linear dependence on concentration. The value of 50 in this equation is only an average figure; it depends on the respiratory quotient and on the ratio of ventilation to oxygen consumption characteristic of the individual.

As mentioned earlier, the presence of CO_2 in the inspired air stimulates breathing, as does anoxia also to a lesser extent. With increased breathing, the ratio of oxygen absorbed to the volume breathed may decrease. The effect of such increased breathing may therefore be taken into account by assuming

$$\Delta PO = 50[1 - (POA/B)](Vo/V) \tag{5}$$

which gives, when inserted into Eq. (2),

$$POL = POA[(B - 47 - PCL)/B] - 50[1 - (POA/B)](Vo/V) \tag{6}$$

In this general equation, PCL will have nearly its normal value of about 40 mm. Hg when any reasonable amount of CO_2 is present in the inspired air and will otherwise have values as given by Eq. (3).

In aviation one is generally concerned with persons who fly from near sea level to high altitudes. The physiologic reactions of such persons are different from those of persons acclimatized to higher altitudes and will depend also on whether oxygen, air, or oxygen-air mixtures are breathed.

Fig. 1 shows the average values of alveolar CO_2 and O_2 pressure in men acclimatized at various pressures or altitudes (lines 1 and 4, adapted from Haldane[1]) and for men taken rapidly from sea level to the indicated pressures or altitudes, breathing air only (lines 2 and 5 adapted from McFarland[3] and approximated as straight lines). The horizontal lines 3 and 6 indicate the normal sea-level values, which may be maintained up to some limiting altitude, by breathing the proper mixture of supplemental oxygen and air. What this limiting altitude is and what mixtures are required will be developed below.

The lines 2 and 5 in Fig. 1 represent transient conditions, in one sense; after some time at a given altitude, the representative points approach the curves 1 and 4 as acclimatization occurs, accompanied by further increase in the normal ventilation rate.

The rapidity with which adaptation occurs varies widely among individuals; a few hours are sufficient for considerable accommodation in some, while days or even weeks are required for others. Because of individual variations it is impossible to derive an equation based on averages that will satisfactorily predict the physiologic state of all individuals under anoxic conditions. However, a great deal may be accomplished with considerable generality, if the aim is changed only slightly. Let it be attempted to derive equations relating oxygen concentrations to altitude, such that one can determine what concentration of oxygen is required at a given altitude (or ambient pressure) to maintain the respiratory equivalent of a given concentration at some other altitude.

Eq. (6) has too many variables as it stands; but, if some relation is assumed between PCL and POL or PCL and B, an equation of the desired form is obtained. The corresponding O_2 and CO_2 curves in Fig. 1 supply three different relations between PCL and POL:

(a) from line 6: No acclimatization,
$$PCL = 40 \tag{7}$$
(b) from lines 1 and 4: Partial acclimatization,
$$PCL = 34 + 0.6POL \tag{8}$$
(c) from lines 2 and 5: Complete acclimatization,
$$PCL = 12 + 0.28POL \tag{9}$$

There is no evidence that anyone fails to adapt to anoxia to at least some degree, so that (a) is clearly an extreme; average individuals may be expected to approximate (b) or (c), depending on the time of exposure to low oxygen concentrations and individual adaptability.

It is probable that for short-time exposure to various altitudes the partial pressure of CO_2 in the lungs does not vary linearly with the ambient pressures; some curve intermediate between the lines shown in Fig. 1 may be more representative for most individuals. However, the straight lines are easier for computation, and the results based on them will serve as limits.

Substituting these three relations in Eq. (6) and putting $POA/B = C$, and from Eq. (3), $Vo/V = PCL/40$, the following three equations result. For the different degrees of adaptation:

$$C = (POL + 50)/(B - 37) \quad (10a)$$

$$C = (1.075POL + 42.5)/(B - 38.5 + 0.015POL) \quad (10b)$$

$$C = (1.35POL + 15)/(B - 44 + 0.07POL) \quad (10c)$$

If C is put equal to 0.21 and the value of B corresponding to various altitudes is inserted, the values of POL in air at selected altitudes can be obtained on the three assumptions. Using the values of POL thus determined, relations between C and B are obtained, giving the concentrations necessary at any altitude to maintain the POL values corresponding to the selected altitudes in air.

For comparison of the extremes, the alveolar oxygen pressures and the highest altitudes at which they can be maintained ($C = 1$) are listed in Table 1, computed on the basis of Eqs. (10a), (10b), and (10c).

TABLE 1

Alveolar Oxygen Pressures at Various Altitudes, and Highest Altitudes at Which They May Be Maintained with 100 Per Cent Oxygen, Computed on the Bases of Eqs. (10a), (10b), and (10c)

Equivalent Altitude in Air (21% Oxygen)	Alveolar Oxygen Pressure (mm. Hg)			Limiting Altitude with 100% Oxygen (1,000 Ft.)		
1,000 Ft.	(10a)	(10b)	(10c)	(10a)	(10b)	(10c)
0	102	102	102	33.8	33.8	33.8
5	75	77	81	37.0	37.0	37.0
10	52	55	64	40.2	40.1	39.9
15	30	37	49	43.5	43.3	43.0
20	16	21	37	46.6	46.4	45.9
25	1.5	8	26	49.7	49.5	48.8

In spite of the great differences in the values of POL computed on these divergent assumptions as to degree of adaptation of the individual, the limiting altitudes are only slightly affected. It may be concluded that any one of the equations given furnishes a good representation of respiratory equivalents—that is to say, for instance, that whatever may be the respiratory state of any individual breathing air at 20,000 ft. breathing pure oxygen will maintain this state at about 46,000 ft.

To compute circulatory conditions and the general physiologic state from respiratory factors requires specific assumptions as to the mode of variation of POL, but this is not the object here.

One point should be mentioned to guard against confusion. The limiting altitudes in Table 1 are seen to be slightly higher for the case where no acclimatization is assumed. It must be remembered that these limiting altitudes are those for which the respiratory condition is the same as for the specified altitude, when breathing air, for the degree of acclimatization assumed. Thus, while the respiratory state of a poorly adapted individual is the same at 43,300 ft. with 100 per cent oxygen as at 15,000 ft. with air, it is clearly not so physiologically favorable as that of an individual acclimatized at 15,000 ft. The unacclimatized person may be severely handicapped, while the perfectly adapted person can carry on normal physical activities under either condition. Even though the latter can maintain his higher POL completely only up to 43,000 ft. according to Table 1, the decrease in his POL in 300 ft. would be only a few millimeters of mercury.

Chart of Oxygen Equivalents

Since the condition assumed in Eq. (10c) is the one toward which average persons tend with time, and particularly since it gives the more conservative limiting altitudes, it has been used in computing the chart of Fig. 2, which shows the concentration of oxygen required at any altitude to maintain the respiratory equivalent of air at sea level and at altitudes of 5,000, 10,000, 15,000, 20,000, and 25,000 ft.

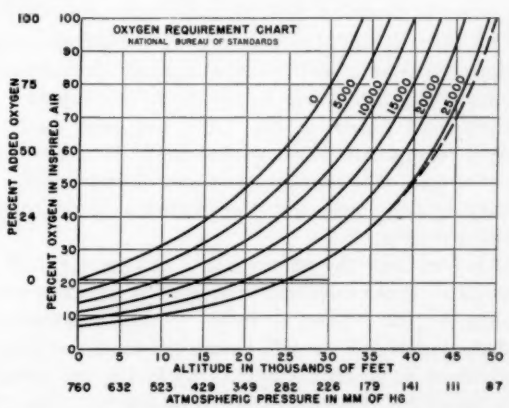

FIG. 2. Oxygen Requirement Chart. Curves show the percentage of oxygen necessary at other altitudes to maintain alveolar O_2 pressures equivalent to breathing air at sea level, 5,000, 10,000, 15,000, 20,000, and 25,000 ft.

The sea-level curve—the one most generally used—is not affected at all by the differing assumptions as to variation of POL and PCL in anoxia, while the other curves would spread out only slightly more on the basis of the other equations, as indicated in Table 1.

For comparison with the other extremes, a broken curve based on Eq. (10a) is shown for the 25,000-ft. equivalent, which gives the greatest spread. The fact that the difference between the extremes is slight is a justification for the approximations made in mathematical representation of the extremes.

No alveolar oxygen pressures are shown on the chart—it may be used to determine the concentrations required to maintain oxygen equivalents regardless of the exact value of *POL* which any particular individual might have under an assumed condition. As pointed out above, the concentrations in the inspired air required to maintain the oxygen *status quo* in the lungs at other altitudes depend only slightly on the exact value of *POL*. Any computation of *POL* values for aviators should be based on Eq. (10b); however, since it gives the more conservative values for this quantity.

From this it must not be concluded that the total oxygen requirements are independent of the *POL* value for the individual—far from it. The increase in *POL* gained by acclimatization or adaptation is accompanied by a considerable increase in ventilation so that the total volume of oxygen breathed is correspondingly increased. If the oxygen in the exhaled breath is wasted, the total oxygen to be supplied is therefore also increased.

On the auxiliary ordinate scale of the chart are shown the percentages of added oxygen in oxygen-air mixtures required to give the desired oxygen concentrations. The total oxygen fraction, C, in the inspired air is related to the oxygen added, expressed as a fraction of the total volume, C_a, by the simple equation

$$C = 0.79C_a + 0.21 \qquad (11)$$

This chart is useful in determining the sufficiency of various oxygen concentrations at various altitudes and in estimating the safety factor in oxygen supply. A similar chart based on Eq. (10b) has been used for several years at the National Bureau of Standards and the Bureau of Aeronautics, Navy Department, in specifying and evaluating the performance of oxygen supply systems.

It is seen that the percentage of oxygen required to maintain a given respiratory equivalent increases more rapidly at the higher altitude. A 50-50 mixture of added oxygen and air will give a sea-level respiratory equivalent at nearly 25,000 ft., while 100 per cent O_2 is required for this equivalent at 33,800 ft. Also, the effective respiratory altitude increases much more rapidly than the actual altitude when breathing oxygen or oxygen-air mixtures. Ascent from 33,800 to 46,300 ft. (12,500 ft.) with 100 per cent oxygen is equivalent to a change from sea level to 20,000 ft., breathing air.

As another example of the use of the chart, suppose that in flight at 25,000 ft. breathing conditions equivalent to 10,000 ft. in air are desired. Following the third curve—for air at 10,000 ft.—up to the intersection with the 25,000-ft. ordinate, the ordinate scale shows that 43 per cent oxygen is needed, which, by the auxiliary scale, can be obtained by an air-oxygen mixture with 28 per cent added oxygen.

Since normal individuals do not lose noticeably in efficiency up to 10,000 ft. in air, with pure oxygen they should be equally able at 40,000 ft. (upper end of 10,000-ft. curve), as far as the effects of oxygen pressure itself are concerned.

To be sure, aeroembolism may occur at this altitude and even cause complete disability, although this is quite apart from the oxygen pressure in the lungs. Also, there may be other effects of reduced pressure which affect the physiologic state, so that one may not immediately identify "oxygen equivalents" with circulatory equivalents, or general fitness equivalents, until or unless experimental data become available to warrant it.

In recent experiments by Dill and Hall[4] on three men breathing pure oxygen at 44,000 ft., arterial saturations equivalent to about 18,000 ft. in air were found. The chart indicates that air at 18,000 ft. is the "oxygen equivalent" of 100 per cent oxygen at an altitude of about 44,800 ft. In the light of the foregoing remarks and the small number of experimental data, no attempt will be made to assess here the significance of this difference.

Effect of Water Vapor and CO_2

In the development of the equations on which the chart is based, inspired mixtures of oxygen and dry air only were implied. This restriction is clearly unnecessary. Since the oxygen is diluted by water vapor and carbon dioxide before reaching the zone of absorption, it makes no difference whether the dilution occurs before or after inspiration. Thus, the addition of water vapor to the dry oxygen-air mixtures indicated by the chart will not change the respiratory equivalent. Likewise, the addition of small amounts of CO_2 (partial pressure up to about 20 mm. Hg) to the mixture will not change the respiratory equivalent if the same partial pressure is added at all altitudes.

The effect of added CO_2 is mainly to increase the ventilation. When breathing air or air-oxygen mixtures this may result in better arterial saturation, as noted by Haldane[1] and by Dill.[5] As indicated by Eqs. (5) and (6), increased ventilation will reduce the loss of oxygen pressure due to absorption (ΔPO), thereby increasing *POL*. Also, the better ventilation may result in more thorough gas exchange in parts of the lungs which are otherwise poorly ventilated. However, in flight at the highest altitudes, anoxia alone causes greatly increased ventilation, ΔPOL is already zero if pure oxygen is breathed, and the partial pressure CO_2 in the lungs is lowered. Under such conditions, breathing in enough CO_2 to raise the partial pressure of CO_2 in the lungs could only result in lower-

ing the effective oxygen pressure. On this basis CO_2 would be of doubtful value. This deduction is in general agreement with the experimental findings of Dill.[4]

Advantages of Acclimatization

As an example of the advantage that a person already acclimatized at a high altitude would have in extreme high-altitude flight with oxygen, consider the state of a person acclimatized at sea level in comparison with that of a person acclimatized at 20,000 ft., when both are supplied with oxygen and taken to, say, 49,000 ft. From Table 1, columns 2 and 3, it is seen that at 20,000 ft. the person from sea level will have an alveolar oxygen pressure of only 21 mm. Hg, as against 37 mm. Hg for the acclimatized person.

But collapse occurs at just about this lower value of *POL;* so with 100 per cent oxygen the person from sea level would still be on the verge of collapse at 46,000 ft., while the acclimatized person would be in his "normal" condition. To go much higher would be fatal for the one subject; the other might ascend another 3,000 ft.—or conceivably even 4,000 ft.—without losing consciousness. According to the chart, the air equivalent of 49,000 ft. with oxygen is 25,000 ft., which is tolerable for persons acclimatized at 20,000 ft.

It is of interest in this connection that the record altitude attained in flight without a pressure suit (Donati, Italy, 1934) was to 47,358 ft. and was apparently very near the limits of the physical endurance of the pilot. It is not recorded whether he had the benefit of any prior acclimatization.

Another theoretic speculation of some interest concerns the oxygen problem in attempted ascents of Mount Everest (elevation 29,000 ft.). On two of six expeditions altitudes above 28,000 ft. were achieved with and without the use of oxygen. No data on alveolar air composition was obtained at this altitude, but a sample taken after several days at 22,700 ft. indicated acclimatization, with alveolar oxygen pressure of 38.8 mm. Hg, and CO_2 pressure of 19.3 mm. Hg, in fair agreement with the curves of Fig. 1.

The chart indicates that such climbers, already acclimatized to 20,000 ft. or thereabouts, could maintain their accustomed respiratory conditions up to the summit by breathing 20 per cent added oxygen, whereas a person acclimatized at sea level would require about 70 per cent, to maintain sea-level equivalent. However, the breathing volume for the person accustomed to 20,000 ft. would be double the value normal for sea level. If the exhaled oxygen were wasted, the advantages of acclimatization in reducing the required oxygen concentration would therefore be somewhat offset. Such considerations are of importance in determining what type of apparatus is best suited to particular uses.

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rectly applicable to airplanes for private use. It is believed that such a plane in the rather near future will present the following characteristics: (1) it will have outstanding vision, (2) it will neither spin nor wind up in a spiral, and (3) it will be stable and consistent in control throughout its entire maneuvering range.

The controls will present no change from present conventional practice that would confuse a normal pilot with conventional training. Yet they will appeal to the average intelligent person who drives a car, who finds it easy enough to handle one more control, but who is confused by any further additions.

The control wheel, looking like the wheel in a car, will steer in similar fashion—whether in the air or on the ground. Its fore-and-aft movement will coordinate all functions of lift. A pedal will provide drag or brake force either in the air or on the ground. The throttle will be preferably a foot type but with means of holding any desired amount of cruising power. For most purposes dual controls will be as superfluous as in an automobile.

At the same time, performance can be stepped up to an extent that will leave no doubt as to the superiority of airplane over automobile for cross-country trips.

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It seems that Newton felt that the real solution of the resistance problem required finding the pressure distribution in the entire fluid space. This he was not able to do since he did not possess the methods needed to go deeper into this problem. He felt that the pressure propagation decreased the resistance—decreased it, in fact, to nil in the case of a nonviscous fluid, as it was found later by d'Alembert. He could not see that the pressure propagation increased tremendously the normal force. He did not consider this case at all; apparently he did not connect the idea of flight with the concept of air resistance.

There are two limiting cases, however, in which Newton's law yields approximately correct results, because the physical picture is similar to that visualized by him. It was shown by recent theoretic and experimental investigations that the lift coefficients of a plane airfoil of very small aspect ratio—i.e., of an inclined strip, whose dimension in the flight direction is large in comparison with its dimension normal to the flight direction—is given with fair approximation by Newton's Eq. (2). In this case, the deflected air mass is approximately equal to the air mass that hits the airfoil.[1]

The second case refers to the theory of drag for shells of extremely large Mach numbers (approaching infinity). In this case, the shock wave is close to the surface of the shell. Therefore, the pressure practically does not propagate into space and the effect of the wind stream is similar to that of an inelastic impact as assumed in Newton's theory.

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